 studies allow me, at present, to make two hypotheses for riddle solving, characteristic of modal musicology.

**The 28 “Quartertones” of Shihāb-a-d-Dīn (19th Century)**

A poet and scholar, Shīhāb-a-d-Dīn Ṭūbīlī Ṭalḥā ibn ʿAbd al-Muḥammad ibn ʿAbd al-Raḥmān al-Ṭūbīlī Ṭalḥā studied at al-Mahāmāt university in Cairo. In a treatise published in 1864, Shīhāb-a-d-Dīn describes a division of the octave in 28 “quarters”. This division attracted many musicological considerations, most of them inspired by Scott Marcus’ opinion that the Sheikh did not really understand the music he described. At least one Egyptian musicologist, on the other hand, tends to agree that the Shīhāb was “the real inventor of the 24-quartertones scale”, and denies the fact that the latter division seemed to be already present in the Middle-East at the time: the Lebanese Mīkhāli al-Mishāqa tells us that he had heard from it in about 1820, in Damascus, from Shīhāb al-ʾAttār.

We will attempt at showing, in the next sections, that Shīhāb-a-d-Dīn’s division is probably a continuation of previous attempts with the Arabian scale, and that his thoughts may have shifted towards a practical way of explaining the music he wrote about.

**Shīhāb-a-d-Dīn’s explanations of the “quarters”**

As a first observation of Shīhāb-a-d-Dīn’s scale, it is possible to say, indubitably, that the Sheikh attests of 28 “quarters” in his treatise.

In his Safīnat al-Mulk the author explicitly counts 28 maqāmāt which are differentiated pitches within the octave, as shown from the lithographic version (Fig. 1):

“and the number of maqāmāt is twenty-eight, divided into usūl and furāṭ; as for the usūl, their number is seven only, and they hold names ordered in ascension [...] and the first is yāḥā’ī and the second dūkā and the third dūkā [sic. See endnote and Figure 2] and the fourth [p. 12] ḥārīkā [or ḥārīkā or gahārīkā, etc.] and the fifth bānīkā [other possible transliterations] and the sixth šāshkāh and the seventh ṣīṭrān [...].”

Shīhāb-a-d-Dīn’s scale is divided into four [four] degrees, and the fourth degree [p. 12] ḥārīkā and the fifth bānīkā, their number is twenty-one and they are divided in three [types] into ‘arābāt, nīmāt of ‘arābāt and ḥārīkā according to the distance [e. the interval] between the degrees, and the proof of this is that the interval [al-bur’d] comprised between two usūl of the seven cited can be complete and is called a burdā, and can be [... p. 14] incomplete and is called a ‘arāb or a nim of ‘arābāt because if you omit a sound beginning with one of the seven usūl and move on [upwards] you cross either the distance of the interval between the usūl and the following degree, and you stop there on it, or you cross [only] half, or a quarter or three-quarters of the distance, and you halt there. By crossing the complete distance and stopping there, you stop on the burdā, and the interval [bur’d] is complete; by crossing half the distance and stopping there, you are on the ‘arāb, and if you cross a quarter
only, you stop on the nīm of the ‘arabā which is its half, and the half of the half is the quarter; by crossing three-quarters of the distance, you stop on the tīk of the ‘arabā and the distance [buď] will be incomplete. In this, the consequence is that the number of the ‘arabāt is seven, as well as the number of the nīmāt and of the tīkāt, and that each of the seven ‘arabāt is between the two of the degrees of the usūl.”

Shihāb-a-d-Din also explains [1864, p.14–15] how the names of the main degrees of the scale evolved and became the ones shown in Figure 3.25

25 Fig. 3. Modern names of the main scale degrees of Arabian music26 (left) and proposed solmisation (2nd column from the right); corresponding Western pitches are shown in the column to the right.27

The author mentions the names of the seven ‘arabāt, which are ZÌNKÌLÅ, KURÌ, BÛSALÌK,28 HÌJÀZÌ, HÌSÅR, ‘AJAM29 and NÀHÄFT.30 Adding to them the tīkåt and nīmåt, which are alterations of the ‘arabåt in the upper or the lower direction, we have the 28 “maqåmåt” of Shihåb-a-d-Dån (Fig. 4).

26 "maqåmåt" of Shihåb-a-d-Dån divided into burdåt, ‘arabåt, tīkåt and nīmåt.31

Fig. 3. Modern names of the main scale degrees of Arabian music (left) and proposed solmisation (2nd column from the right); corresponding Western pitches are shown in the column to the right.

Fig. 4. The “maqåmåt” of Shihåb-a-d-Dån divided into burdåt, ‘arabåt, tīkåt and nīmåt.
However, this contradicts the Modern Arabian theory of the “quarters”, since 24 quartertones in an octave can not sum up to 28 “quarters”, unless the “quarters” are not equivalent to “quartertones”.32

As a conclusion to this point, let us note that Caron and Safvate (among others), in their retrospective study of the music of Iran,33 underline that Shihāb-a-d-Dīn’s division corresponds to the division of the seven main intervals of the Arabic scale, commonly stated in modern literature as composed of one-tone and three-quartertones intervals, in further halves and quarters of “tone”, regardless of the values of the “tone”.34 (Fig. 5).35

**The way to 28**

Nowadays, the basic scale of Arabian music is usually given as a two-octave scale composed of one-tone and three-quartertones intervals (Fig. 6).

![Fig. 6. Basic scale of the Arabian music according to Erlanger36. B stands for “half-flat”.37 The scale of Figure 7 corresponds to the first ascending octave on this figure.](Image)

It is important to keep in mind that the equal-quartertone division was implemented very late in the history of this music, under the influence of the Western 12-semitone scale.38 Moreover, the equal-quartertones scale is far from corresponding exactly to the intervals used in the performance of Arabian music.39 As a matter of fact, Arabian musicology still fails to determine which were the actual intervals used in early Arabian music, albeit clues exist in the specialised literature.40

Before we examine these clues and use them to explain the formation of Shihāb-a-d-Dīn’s scale, let us have a closer look at the scale which can be deduced from the anonymous A-sh-Shajara41 and from the (a-š-) Šafadi epistle. In these treatises,42 the authors also use the concept of burdāt for the main degrees of the scale, but also anšāf, which are “halves” of the intervals.43

The resulting scale, expressed with nowadays Arabic names of the degrees, is showed in Figure 7. The scale comprises seven so-called “tone-intervals” (but the “tone” is undefined) with their “halves”,44 which amounts to 14 intervals (“halves”) per octave.

![Fig. 7. The basic scale of Arabian music from G to g (left to right), with the seven isāl or burdāt (blue and orange ovals in succession above standing for G a b c d e f g – “hf” stands for “half-flat”) and the anšāf (green ovals below G a b c d e f g or a b c d e f g). The names of the upper degrees are, from left to right, YĀJĀK “USHAYRĀn ‘BĀQ RĀST DŪKĀ SIKā JAHĀRKA and NAWĀ; the lower notes are (same direction) qarār-HIṢāR qarār ‘AJAM KAWSHT ZIRKūLĀ KURD BūSAlIK and HLIJZ (or SABB).](Image)

The study of the (a-š-) Šafadi epistle shows that this division might have been too rough for his time, as the author uses a differentiation between “upper half” and “lower half”.45 This could have meant a division in three of the “tone-intervals”,46 or a further division in two of the “halves” resulting in “quarters” which would correspond to Shihāb-a-d-Dīn description of the scale.

However, the remaining question is to know what the nature of these “quarters” is, and how we can fit 28 “quarters” into the 24 quartertones one octave can normally contain.

**Fitting 28 in 24**

Let us first stress that the Rāst scale shown in figures 3 to 6 is quite recent in Arabian music, and that the first Arabian philosophers and theorists, mostly inspired by Ancient Greek theories,48 used scale constructions49 which favoured a regular perfect-fourth tetrachord + perfect-fourth tetrachord50 build of the scale based on the tuning of the ‘ūd in fourths (Fig. 14), resulting in what is today commonly known as the scale of the maqām Yākā (Fig. 8, Fig. 9).51

As I have argued elsewhere,52 I strongly suspect that an equal string-parts construct (Fig 14) was the originating point of the Arabian scale; 9th and 10th century polymaths such as (al-) Fārābī and (ibn) Sinā, who gave detailed theoretical explanations on the Arabian scale of their time, used equal-division of the string together with the Pythagorean so-called “diatonic” division in their description of the positioning of the fingers on the neck of the ‘ūd.
usually tuned in alternated fifths and fourths, were in intervals are at the core of modern Arabian and Egypt at that time) was the demonstrations (Fig. 13).

Important to date Arabian Modern theorist, Mīkhāʾīl used in music theory and teaching musical instrumentarium. We also know that the most other long necked lutes, such as the Byzantine music theory and teaching.

There is however to date no proof that this equal music, and seem to be present from the origins of this in most of the Western specialised literature called particular.

Nonetheless, we know that at the time Shihāb-a-d-Dīn wrote his treatise equal division of the string was in use in music theory and that the favoured string instrument of the Ottoman empire (which still ruled Egypt at that time) was the tūnbūr, (including for Byzantine music theory and teaching – Fig. 10) and that other long necked lutes, such as the nashʿat-kār (Fig. 11) usually tuned in alternated fifths and fourths, were in favour at that time and later (Fig. 12), besides evidently the introduction of the European violin in the Arabian musical instrumentarium. We also know that the most important to date Arabian Modern theorist, Mikhāʾīl Mashāʿqa, used the tūnbūr for his theoretical demonstrations (Fig. 13).

Fig. 8. First of two possible representations of the maqām Yākā scale commonly ascribed in contemporary literature with a lower (to the left) rāst tetrachord (14 3 3 – the “↑” is used for highlighting the ascending direction of the intervals) on G, and with an upper bayāt tetrachord (13 3 4 beginning on d). 53

Fig. 9. Second of two possible representations of the maqām Yākā scale; this may be the original construct of the scale, with two conjunct rāst tetrachords (on G and c) completed (for the octave) by a “one-whole-tone” interval (between f and g).

The equal string-parts construct was used in particular to determine finger positioning of the so-called “zalzalian” intervals, known as “neutral” intervals in most of the Western specialised literature; these intervals are at the core of modern Arabian and maqām music, and seem to be present from the origins of this music. There is however to date no proof that this equal string-part construct was used as the main basis for the division of the octave, although strong hints of its presence are present in the early Arabian literature on music.

Nonetheless, we know that at the time Shihāb-a-d-Dīn wrote his treatise equal division of the string was in use in music theory and that the favoured string instrument of the Ottoman empire (which still ruled Egypt at that time) was the tūnbūr, (including for Byzantine music theory and teaching – Fig. 10) and that other long necked lutes, such as the nashʿat-kār (Fig. 11) usually tuned in alternated fifths and fourths, were in favour at that time and later (Fig. 12), besides evidently the introduction of the European violin in the Arabian musical instrumentarium. We also know that the most important to date Arabian Modern theorist, Mikhāʾīl Mashāʿqa, used the tūnbūr for his theoretical demonstrations (Fig. 13).

Fig. 10. Chrysanthos of Madytos’ depiction of a “tūnbūr”, used in his explanations about Byzantine music, for theoretical purposes. 60

So what if Shihāb-a-d-Dīn based his division of the octave on such an instrument tuned in alternative fifth(s) and fourth(s)? 61 The answer lies in Figure 15, which shows such a construct with a division of the octave on two strings “tuned in fifth in 28 “quarters”, or equal string-parts, the upper fifth being divided in 16 equal parts and the lower fourth in 12 equal parts, which together amount to 28.

The main degrees are obtained from a division of the string in 12 equal parts; then these parts are divided in two in order to obtain the akhḍ or “halves” of tones (which are now defined, but different from one another), with these halves divided in turn in two parts which give 4 “quarters” in one “tone”, which multiplied by 7 (main “tone” intervals) amount to the 28 “quartertones” of Shihāb-a-d-Dīn.

We can deduce from this hypothesis that the resulting main scale of Arabian music in the time of Shihāb-a-d-Dīn would be d e f g a b d’ (or 13 3 4 4 3 3 4 in standardised “quartertones” – the “↑” is for highlighting the ascending direction of the scale) which, if started on its fifth scalar degree and then transposed down an octave (starting on G) gives us the scale of maqām Yākā shown in Figure 9, with a standardised 14 3 3 4 3 3 4 (in quartertones) scale. 62

In the Shihāb-a-d-Dīn’s construct as I propose it in Figure 15, however, the so-called “three-quartertones” intervals of the modern Arabian theories are conceptually closer to Saḥfy-a-d-Dīn al-Urmawi’s mujannabat, 63 with a “small mujannab” between d and e’ on the first string (approx. 151 cents), and between a and b” on the second, and a “great mujannab” between e’ and f on the first string, and between b” and c’ on the second (approx. 165 cents). The next intervals on the strings are the “minor tone” (approx. 182 cents) and the Pythagorean tone (approx. 204 cents).

This construct gives us a good idea of the process of the octave division and a good explanation for Shihāb-a-d-Dīn’s scale, but what if some maqām musicologist insisted that maqām Rāst scale is the only scale on which the Arabian general scale may be based?

The answer to this question lies again in starting the Yākā scale of Figure 9 a fourth higher (beginning on c), which gives us the typical 14 3 3 4 4 3 3 scale. Another clue for this is the fact that a common tuning of the ḫūd in the Middle-East today uses a supplementary string in the lower part of the scale, which is frequently tuned a major (or Pythagorean) tone lower than the next string, with results as a tuning in G A d g c’ and f’. 64 Including the intermediate degrees b’ and e’ on the A- and d-tuned strings of Figure 14, we obtain a 14 3 3 4 4 3 3 scale which is the maqām Rāst.

This way of thinking gave me a clue as to the problematic of the 22-shruti scale that I shall explain in the next sections. 65
Fig. 11. A nashʾatkār made 1928 in Damascus by the famous lute-maker Anṭūn Nahḥāṭ.66

Fig. 12. The quality and production certificate (glued in the inner part of the belly) of the nashʾatkār in Figure 11.

Fig. 13. Mashīqa’s division of the string of the ṣuntār explaining how to establish an equal-division of the octave in 24 quartertones.67
Fig. 14. The 12 equal string-parts construct based on a tuning of the ʿud in fourths. 68

Fig. 15. An explanation of the construction of Shihāb-a-d-Dīn’s scale in 28 “quarters” on the example of a “lute-type” string instrument tuned in fifth (or alternate fifths and fourths). 69
A HYPOTHESIS ON THE FORMATION OF THE 22-ŚRUTIS SCALE

Twenty years ago, it would have been most unlikely that I would study Indian music. Fifteen years ago, and as I started reading about the theory of this music, it seemed an impossible task as there was only very little accessible material. Ten years ago, I gave up on the śruti system and decided to concentrate my efforts on Arabian music: there were enough riddles with this music to keep me and other musicologists busy for a few more decades.

However, and as there was more available specialised material due to digital archiving of scientific reviews and early books, I finally came across materials which, along with my continuous efforts to unveil Arabian or Byzantine early books, I finally came across materials which, along with my continuous efforts to unveil Arabian or Byzantine early books, I finally came across materials which, along with my continuous efforts to unveil Arabian or Byzantine music theory riddles, triggered, at last, the solution that I am proposing in the present. I am first indebted to musicologists and writers, mainly Ananda Coomaraswamy and Nazir Jairazbhoy, and to Shihāb al-Hijāżī whose 28 “quartertones” I believe to be a recent replication of a concept dating to the dawn of the literate period. I am also greatly indebted to Ṣafīyya-al-Dīn al-Urmawi whose scale with two unequal mujannab kept me busy for a long time before I could explain the fact that two intervals may be rather different in size, though conceptually equivalent.

The conceptual similarities between the Indian śruti scale and the Arabian scale

To start with, I must confess that I always thought that the Indian and Arabian Art music were based on similar general rules, but that nationalism, Western influence, the different languages or simply the superficial cultural differences were preventing them to be compared.

The main influence of Western musicology was to be found in the avoidance of references to any possible existence of “neutral” intervals in Indian music (theory) and in the generalized use of Pythagoreanism in order to explain the scale and the intervals. It is mainly through the effect of this musicology on local theorists that we may explain the stress on the octave scale as a whole, and not on smaller scalar, if not melodic units, i.e. the fourth and the fifth. Moreover, the Western tendency to idealize music as a science, a concept which spread very quickly among local musicologist (whether Arabs, Turks, Greeks or Indians) enlisted in a competitive race trying to give their music a “respectable” basis (that is “Pythagorean”). A corollary to this is that music performance lost its role as the basis for theories, and that music theories à la European became the rule, apart from imposing normative instructions to the performers.

As a result of the refusal of the above influences, the hypothesis for the formation of the theory of the 22 śruti I propose is based on one deduction and two assumptions.

- Firstly: the deduction to which I came after studying the internal composition of the “classical” śruti scale is that the Ṣ4 Ṣ2 Ṣ4 Ṣ3 Ṣ2 śruti division found in Bharata-muni’s Nāṭyaśāstra is, conceptually, very similar to the Arabian so-called “sazalian scale,” notably expressed by Ṣafīyya-al-Dīn al-Urmawi in the 13th century.

- Secondly: the first assumption I made was that the two Indian and Arabian general scales are not only conceptually equivalent, but that the original “tones” (i.e. the 4, 3 and 2 śruti “tones” and the 4 and 3 “quartertones” of contemporary Arabian theories) should also be very close in practice, if not in theory. The assumption is that these intervals should be approximately the same in the original Indian music and Arabian music, and that the resulting theoretical intervals should express, at least in their proportions, the actual differences in sizes of the intervals used in performance.

- Thirdly: the second assumption is that the Indian musical scale is originally based on the fourth, and that the vīnā tuning should be the key to the solution.

As the reader can deduce from these assumptions, I have tried to apply my knowledge of Arabian theories of scale formation to the problem of the construct of the śruti scale, with the results that I discuss below.

Is the number of śruthis equal to 22 in an octave?

The śruti system has challenged scholars for centuries, some of them discussing and disputing even the number of śruthis in the octave, as Kolinski puts it:

“[I]t is necessary to discuss the actual meaning of the allocation of twenty-two śrūtis within one octave. For the supporters of the divisive concept it has been an easy task to arrive at the required number of śrūtis. Hornbostel and Lachmann have attempted to trace the origin of the system back to a hypothetical instrument supposed to be related to the Chinese k'ìn. After a whole series of alterations of the actual fingerboard of the k'ìn the two scholars finally arrive at a hypothetical fingerboard of Bharata's vīnā which in fact includes twenty-two śrūtis within the octave […] but a similar method would allow one to establish also any other desired number of śrūtis. Fox, Straungways approaches the problem in a different way: he projects all 14 murchanas, that is, the whole of the theoretically possible modal varieties, mentioned by Bharata, into the same octave, but, unfortunately, gets only
twenty instead of the twenty-two expected śruti; so he adds the lacking two śruti 'by analogy.' Danielou [sic], on the contrary, was forced to eliminate one śruti when his calculations led him to the number of twenty-three. Finally, Clements' painstaking calculations yield twenty-five śruti within the octave. This time, however, it is Bharata himself and the other old Indian theorists who are accused of having made a mistake, and Clements insists that the real number of śruti is not twenty-two but twenty-five. Still, the majority of the all-Indian Musical Conference has voted in favor of the consecrated number of twenty-two śruti within the octave.

Let us first note that the śruti is not an interval used as such in the scale, but should be considered as a "quarterton" of Modern Arabian theory, and as a component of such intervals, as Coomaraswamy writes:

"The scale of twenty-two notes is simply the sum of all the notes used in all the songs—no musician sings a chromatic scale from C to [c] with twenty-two stopping places, for this would be a mere tour de force. The 'quarterton' or śruti is the microtonal interval between two successive scale notes: but as the theme rarely employs two and never three scale notes in succession, the microtonal interval is not generally conspicuous except in ornament." 80

Let us also note that Kolinski, among other scholars, favours the harp-type vīnā hypothesis (and the cyclic one) and bases his argumentation on Coomaraswamy's article "The parts of a Vīnā." 82

The simple solution that we propose for the śruti scale formation is based on the (complementary) assumptions that the number of śruti within one octave is effectively 22, and that the instrument cited in Bharatamuni's Nāṭyaśāstra is a lute-type vīnā (Fig. 18, Fig. 19). 93

The "small" Indian tones and Urmawi's mujannabāt

As I learned some time ago about performing maqām Ṣahāb with my teacher and friend Saad Saab, 94 I came to the conclusion that not only the placement and intonation of the sīkā and the ʿIRĀQ degrees, equivalent in Middle-Eastern maqām theories to the Westernised es and bs, are subject to changes according to the organology 95 and instrument making, 96 morphology 97 and maqām type, but that there are also different positionings for (for example) the es 98 degree according to the family type of the maqām or of the tetrachord. 99 In concrete terms and to put it simply, the es degree is much closer to "natural" 100 e in the rāst tetrachord than it is, for example, in the bayātī tetrachord, although both positions are considered to correspond to the sīkā (es) degree.

As a result, in current practice the "neutral tones" or mujannabāt 101 in Arabian music are around 170 cents for the first, greater mujannab M1, and around 125 cents for the smaller one M2. The "one-whole-tone" interval is usually played at about 200 cents.

In the basic maqām Rāst of Arabian music, the scale may be (7) T M1 M2 T T M1 M2 where T stands for "one-whole-tone", M1 stands for "first (or greater) mujannab" (which is smaller than the one-whole-tone, but bigger than a semitone, whatever the latter may be), and M2 stands for "second mujannab," or "smaller mujannab" (which is smaller than the first mujannab, but still greater than one "semitone" interval). These mujannab intervals fit, conceptually if not in measurements, with the description given 102 by Ṣafyy-a-d-Dīn al-Urmawi about the two forms of mujannab, a "greater" one made out of two limma intervals, and a smaller one made of one limma plus one (Pythagorean) comma (Fig. 16 – above). Urmawi, however, in his handling of tetrachords in Arabian music, uses the generic letter "c" for the mujannabāt thus eluding theoretical differences between (L C) and (L L) as shown in Figure 16 (above); the intervals composing the mujannabāt intervals (Fig. 16 – below) are also approximately equal, which gives a supplementary indication in favour of an undifferentiated use of the two forms of the mujannab in (theoretical) practice 103, 104

![Fig. 16. Urmawi's use of the mujannab: (above) explanation of the theoretical values of the mujannabāt; (below) excerpt from a Ms. of the Book of Cycles by Urmawi showing the undifferentiated use of the mujannabāt by the author (the letter "c" – or "b" – above the two first intervals from the right) for the ʿIRĀQ genre (or tetrachord): both intervals "c" (for "mujannab") contain (roughly) equally divided smaller "elementary" intervals which compose them.) 105

In most of the current Arabian Middle-Eastern theories, the two forms of mujannab are also considered as equal, theoretically, and are equated to the three-quartertones interval, with the maqām Rāst scale expressed as f4 3 3 4 4 3 3 (in equal quartertones). However, M1 and M2 belong to early Arabian theory, and to nowadays (and yesterday's recorded) Arabian music 106.
It is difficult to equate the $14334433$ (in theoretically equal quartertones) maqam Rāst scale in this form to the śruti main scale $14324432$ (in śrutis) found in Bharata-muni's treatise of music, equally as difficult as to equate the 24-quartertones scale with Shihāb-a-d-Din's scale made up of 28 "quarters".

However, considering performance practice that the $TM_1M_2TM_1M_2$ maqam Rāst scale is the closest to the intervals in praxis, we can compare this scale to the basic śruti scale $14324432$ (in śrutis - see Figure 17 for an alternate formulation), on the basis that $T' = 4\text{ śrutis}$, $M'_1 = 3\text{ śrutis}$ and $M'_2 = 2\text{ śrutis}$. The two scales become thus conceptually equivalent, and the intervals are conceptually identical, if not (approximately) equal: the latter is the first of the two assumptions we made above; we examine the second assumption in the next section of this article.

\[
\begin{array}{cccccccc}
3 & 2 & 4 & 4 & 3 & 2 & 4 & 4 \\
\text{sa} & \text{ri} & \text{ga} & \text{ma} & \text{pa} & \text{dha} & \text{ni} & (sa) \\
4 & 3 & 2 & 4 & 4 & 3 & 2 & 4
\end{array}
\]

Fig. 17. The sa-grama scale as explained by Kolinski. The numbers in the lower row express the values of the leading (and ascending) intervals to the tone; the number in the upper row the values of the intervals between two successive tones: both interpretations have been challenged, although the $14324432$ version seems to be today predominant.

Two assumptions and one instrument

As I was undergoing research for my first book on Arabian music theory and practice, the preponderance of the 'ūd in early theories led me to undertake a detailed study of the origins of the instrument. The result was that the first appearances of the short-necked (and unfretted) lute, either in the iconography or in literature, were situated beyond any doubt in Ancient India, close to the beginning of the first millenary a.d. This predates the Islamic-Arabian 'ūd, and the lute-vīnā (Fig. 18, Fig. 19) is probably the direct ancestor of the Persian barbat, which seems to have been an early form of the 'ūd.

Short-necked lutes are commonly unfretted instruments, offering versatility for interval sizes. It is often difficult to make precise measurements in order to determine fret positioning, or even to draw accurately fret marks on the finger-board. Halving a length is easier, with a... string.

Should we wish to determine, for example, the position corresponding to the fourth of a vibrating string, it suffices to fold an equal length string twice, and measure with the folded string the distance from the nut.

In Early Arabian music, further (equal) division of the string-part corresponding to the perfect fourth would be an easy task, and would result in the successive tetrachords depicted on Figure 14. In the previous sections, we saw that further divisions of the resulting "tones" lead to a satisfactory interpretation of Shihāb-a-d-Din's scale.

The 22-śruti scale as an equal-division construct

The division in 9 equal parts of the string-part corresponding to the perfect fourth on the lute-type vīnā is shown on Figure 20.

Dividing the first string (the upper string tuned in $G$ in Figure 20) into 36 equal parts (i.e. of equal length), with 9 parts to the (perfect) fourth, we obtain a division which reproduces the śruti spread in a perfect fourth as described in the Bharata-muni's Nāṭyaśāstra, i.e. three "tones" in the fourth, with corresponding numbers of śrutis 4, 3 and 2. Let us call the first tone, with the 4 first śruti (equal parts of the string), "Pythagorean," as its length ratio is 8/9 [as $(36-4):36 = 8/9$]. It measures 204 cents.
We may decide to name the second “tone” “first (or greater) mujannab”, similarly to the Arabian theory. This first mujannab is made up of 3 śrūtis (equal parts of the string divided in 36). It is theoretically equal to 170 cents. As for the second mujannab, it is made up of two śrūtis (idem) and equates to 124 cents. We find equivalent “tones” in the second string, for the second perfect fourth (cf). We thus obtain 6 degrees of the scale, containing successively 4 3 2 4 3 2 and 2 śrūtis. The remaining “one-whole-tone” is obtained on the third string, with the resulting 1 4 3 2 4 3 2 4 scale (in śrūtis).

Thus, śrūtis are at the same time “equal” (as equal parts of the string) and “unequal” (as intervals measured by modern scientific methods). Their exact value varies between 49 and 63 cents, with an average value of 55 cents. This could explain why śrūtis are considered as equal in Early Indian writings on music and that these śrūtis are taken as unequal.

Furthermore, transposing any scale in such a division of the octave would result in small discrepancies due to the different sizes of the śrūtis depending on their position on the fingerboard; this probably means that this division was taken as an indication for the effective positions of the fingertips on the fingerboard, and that this fingerboard was, consequently, not fretted.

Short discussion about the outcome

One of the objections to the theory of the 22-śrūtis scale as an equal-division construct could be that the 1 4 3 2 4 3 2 4 scale is different from the 1 4 3 2 4 3 2 4 scale (both in śrūtis), and that the scale deduced from Figure 20 begins on pa (G) and not on sa: my answer would be that the most important feature in this scale is the composition of the fourth, which is 4 3 2, and that by combining a fourth (4 3 2) with a one-whole-tone (4) and another fourth (4 3 2), we obtain sa 4 ri 3 ga 2 ma 4 pa 4 da 3 ni 2 sa.

Let us also remember that:

- Transpositions in fourths (or fifths) are frequent in melodic music, mostly whenever string instruments tuned in fourths (or fifths) are used in performance.
- Arabian lutes have a “zero” string that is very frequently (commonly) tuned in a “one-whole-tone” step with the first (second) string, which gives us, if we start the scale a “one-whole-tone” lower, 1 4 3 2 4 3 2 (in śrūtis). By starting the scale on the fifth degree we obtain sa 1 4 ri 3 ga 2 ma 4 pa 4 da 3 ni 2 sa.

Again, the octave is irrelevant in this matter since the fourth seems to prevail in Early and more recent modal music.

Now, with regard numbers 9, 4, 3, 2, and others that can be deduced from Figure 20, such as 36 (equal-parts of the string) and 29 (as the numerator of the 29/36 ratio of e° and b²) what can be said? There may be religious or cultural explanations for those numbers, but I cannot resist the urge to propose another, simple explanation for their use in the construct.

I have explained elsewhere how numbers of small intervals composing a bigger conceptual interval can be used as qualitative markers for these intervals; in Šafiyya-ald-Dīn’s theory of the scale, for example, the two mujannab(s) each host two intervals (Fig. 16), although the exact theoretical measures of this two-form interval are different. Conceptually, however, the two forms of the mujannab are considered by Šafiyya-ald-Dīn (al-) Urmawi as being the same interval.

If we think of the numbers of śrūtis amounting to an interval in the 22-śrūtis scale, the 4, 3 and 2 clearly define different intervals, conceptually, although the exact measures of these intervals may slightly vary because of the positions of the śrūtis on the fingerboard of the vina. The question arising in this case is “why not begin with number 1 and use 3 2 and 1 for T M₁ and M₂?” The answer to such question can be given in three argumentative parts:

- Firstly: the numbers of small intervals composing the greater conceptual intervals must somehow reflect the actual sizes of the intervals; in this case, 124/204 as a ratio of cents value between the smallest “tone” (the “small mujannab” M₁) and the “Pythagorean tone” of the 22-śrūtis scale (Fig. 20) is closer to 2/4 (or 1/2) – which is the ratio of the śrūtis contained in the corresponding intervals – than 1/3 in the other version.
- Secondly: the 4 3 2 division of the fourth gives a perfect match for the Pythagorean tone as the result of the ratio 8/9, whenever 3 out of 24 divisions in all in the “3 2 1” partition results in a 7/8 ratio.
- Thirdly: the 29/36 ratio (374 c.), which may seem awkward at first, is a close match to the much simpler 4/5 ratio (approx. 386 c. and a practical way of approximating the latter.

Another question that may arise is “why then only 9 śrūtis in a fourth and not 10, 12 or more for more accuracy?” There is a much simpler answer to this question: the main aim of this division of the octave was not normative, but indicatice. In other words, the first objective of such a division would be to give simple qualitative indications to the performer as how to perform a mode, and not to determine the exact sizes of the intervals in use.
CONCLUSION

The equal division of the string is a plausible hypothesis for some of the scale constructs found in the maqām and other forms of modal music.

In this paper, I give two of these constructs, one of which is a full illustration of Shihāb-al-Dīn’s “28-quarters” scale; in the case of the 22-śruti scale, further research is needed in order to determine whether the equal string-division may give answers and clues beyond the discussion undertaken here.\(^{134}\) If such a hypothesis receives confirmation with Indian music, it would be legitimate to postulate that the introduction of the Western concept of “neutral” (i.e. “foreign”) and “equal” intervals alongside (ironically) with the use of Pythagorean and just intonation concepts, and the evolution of concepts that ensued, transformed the Early indicative and conceptual theories into normative and measuring theories.\(^{135}\)

This explains how the intervals which were in use until recently in the history of modal music were approximate\(^{136}\) tended to become fixed-sized intervals.\(^{137}\) The normative trend represented by either equal-temperament or Pythagorean incantations to “science” prevent today most musicologists from understanding the basis from which early theories are built. This leads to very complicated explanations on phenomena which could well be, in substance, quite simple: these theories were mainly, if not all, conceptual in their essence, especially in the absence, in Ancient times, of accurate means of measuring intervals sizes.\(^{138}\)

It suffices however to put aside Western misconceptions about modal music in order to find clues about early (or less early) theories, and to determine how they were distorted in the West, then afterwards or in parallel, in autochthonous modern musicology.\(^{139}\)

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**Fig. 20.** Construction of the śruti scale based on a division in 9 equal parts (śruti) of the fourth of the strings of the lute-type vīnā.\(^{140}\)
Bibliography


13. BEYHOM, Amine: “Intervalle de mesure ou intervalle conceptuel? Les errements des notations et théories des musiques arabes” [Université Paris Sorbonne, 2006a-4-1].


15. BEYHOM, Amine: “Approche historique de la relation dialectique entre théorie et pratique chez les théoriciens arabes; A Historical approach of the dialectical relation between theory and practice in the writings of Arabian theorists”, Le Congrès de musique arabe de 1932 au Caire, 75 ans [الجرس | Le Caire, 2007a-3-11].

16. BEYHOM, Amine: “Les tribulations du hijāḍ autour de la Mare Nostrum” [Université Paris Sorbonne, 2007b-3-24].


22. BEYHOM, Amine: Une approche systématique et diachronique de la modalité maqâmiyye, Habilitation Thesis, Université Paris Sorbonne [Institut Finlandais à Paris (France), 2010b-3-9].


30. CHRYSANTHOS (DE MADOYES) and Panagiotis G. PELOPÈDES: Θεραπεύειν Μεγάς του Μουσικού [Theorίκην mega tis mouσiκhs – Great theoretical book of music], En Tergeste: ek tez typographias Michael Vais (Michele Weis) [Trieste - Italie, 1832].

31. CHRYSANTHOS (DE MADOYES) and Kaitê RÔMANOU: Great theory of music by Chrysanthos of Mydoytos, Thesis, Indiana University [1973].

32. CLEMENTS, Ernest: Introduction to the study of Indian music, Longmans, Green and Co. [1913].


35. DANIÉLOU, Alain: Introduction to the study of musical scales, The India society [London (printed in Benares), 1943].

36. DANIÉLOU, Alain: Northern Indian music: theory and technique Vol 1, C. Johnson [Calcutta, 1949].

37. DANIÉLOU, Alain: Traité de musicologie comparée, La nature de la musique, Hermann [Saint-Étienne – France, 1959].


Notes

1 “Unveiling the repetitions of the scholars [another meaning for akhbar is priests] in explaining the modes.” The three parts of the title are built in rhymes, as were often titles of poems and musical treatises in Arabian musicology. Kashf al-Asrār, or “Unveiling the secrets (of) is a common beginning of titles of poems and treatises found in a wide panel of Arabian writings. The first part of this article is a tribute to my predecessors, mainly in the 19th and the 20th century, who “unveiled” much. Karkar WTF “repetition”, and finds its contextual meaning with the following word akhbar, which means “prelates”, “scientists”, “scholars” – see [Ma’lūf, 1997, p. 678 & 113]. Most of the authors of the music “riddles” in modal music were prelates, who also happened to be scientists and scholars: these Uhr-musicologists have repeated the same or similar pattern of constructing the scale, albeit with differences in the details! As for ta’wil al-adwr, the first term means...
“explanation, interpretation, connected with” – see [Maulûf, 1997, p. 21], and adastro is used, at least since Safit-y-a-d-Din [Umawi (al-), 1880; 1882; 1884; 1938] in the 13th century, as the original description of what could be called today a “modal scale” (an accurate study of the different ways of describing scales in relation to their modal characteristics is planned in [Beyhom, 2014, forthcoming]). As a result, the title of this article is a tribute to all those, often prelates and scholars with cheerful spirit, who tried to explain, elucidate or transmit the problematic of the modal scale. As Ernest McClain affectionately put it in a recent correspondence [2012], “[the foundational attitudes of Rq Veda scrits is one of great amusement with themselves; it was pure genius to describe themselves as Holy Priests as ‘frogs croaking around a pond’].

2 “By way of introduction”. I would like here to thank François Picard, Professor at the Université de la Sorbonne and Scott Marcus, Professor at the UCSB, both being members of the Academic Board of NEMO-Online, for their useful (and numerous for the latter) comments, corrections and suggestions. I would equally like to thank Richard Durnbrill, our most respected administrator, for the huge amount of time he spent correcting my English.

3 Plural of urjāzā, a poem in the rajaṣ (corresponding to mustafīlān six times) prosodic meter ([Abdelnour, 2008, p. 62 & 69], [Maulûf, 1997, p. 250]).

4 I use the plural for śruti with an “s” (śrutis), for reasons of convenience.

5 For example the l-o-g-o scales in my thesis [Beyhom, 2003b, p. 230–234 & 269–283], with variable numbers of intervals to an octave.


7 Whose writings I could read only in some European languages.

8 Mainly Pythagorean as I have tried to show in [Beyhom, 2010c], and as I plan to further demonstrate in [Beyhom, 2014] (forthcoming).

9 In the meanwhile, the publication of my first book and the foundation of a research centre on Arabian music took also a lot of time and energy.

10 The habit of differentiating music(s) whose general (and most important) features are so close that they should be originated from the same point: I document the influence of this nationalist attitude on the music in different regions of the maqâm realm in [Beyhom, 2014] (forthcoming).

11 Our main sources for Shihåb-a-d-Din’s biography are [Zirîkî, 1980, v. 6, p. 38] and [Shiloh, 1979, v. 10, p. 327–328].

12 A Muslim title for religious dignitaries, commonly translated as “Sheikh”.

13 Born in Mecca, in the HÎjaz (Arabia).

14 There are a few manuscripts predating the published version of Shihåb-a-d-Din’s treatise [Hîjazi (al-Malaki al-), 1864], of which we know [Hîjazi (al-Malaki al-), 1843; 1855]; the latter are sometimes more accurate and were helpful in correcting typographical mistakes in the printed book as shown in Figure 2.

15 See for example [Marcus, 1989, p. 71–72].

16 This was for example the claim of Fathi (al-) Khamsî, an Egyptian musicologist, during a conference organised by the Arab Academy of Music (AAM) in Cairo in 2007 [The 1932 Congress on Music in Cairo, 75 years (A critical approach of the 1932 Congress on Arabian music held in Cairo in 1932)]. This musicologist kept speaking in his conference about the “24-quartettones scale of Shihâb-a-d-Din,” and refused to answer our questions (with a few colleagues) about the fact that Shihâb-a-d-Din clearly defines a 28-step scale in his treatise (the 28 “quarters”) and not a 24-step scale.

17 Mashâqa has wrongly been ascribed as the “inventor” of the 24 quartettones scale common nowadays in the Middle-Eastern music theories: for a discussion on this matter and the fact that al-’Aţîf, as Mashâqa ascribes it, was well aware of this division, see Romeva’s argumentation in [Mashâqa, 1899b, p. 4–5], or our [Beyhom, 2014] (forthcoming).

18 Also called maqâmî (plural of maqâm).

19 I use the following conventions for Arabian notes, tetrachords and modes (maqâmî) names: a note name is written in capital letters, such as for the degree RĂST (equivalent to the Western C in the Arabian contemporary theories of music); a tetrachord name is written in small letters, for example nār for the tetrachord composed in the same theories) of three successive ascending intervals comprising 4, 3 and quart tones each, as for the maqâm Rāst. This differentiation is helpful in such cases when all three RĂST note-degree, nār tetrachord and Rāst mode bear the same name (please refer to the introductive part of my first volume on Arabian music [Beyhom, 2010c, v. 1, p. xvi–xxiv] for detailed information about the reasons underlying the use of YĀKĀ instead of YĀKĀH for example, or for other peculiarities of the transliteration).

20 This should be “SĪKĀ” as in the Ms. z 2935 (Fig. 2).

21 The author uses here the terms maqāf al-’Arâd, which means “the distance of the interval” or, in another interpretation, “the distance corresponding to the interval”.

22 [1864, excerpts from pages 11, 12, 13, 14 and 15].

23 This lithographic version is referenced as II 1864 in this article.

24 [Hîjazi (al-Malaki al-), 1864, feuillet 4 vê].

25 The story of this evolution is complex and ascribed in [Beyhom, 2014] (forthcoming): we use in the following figures the standard contemporary names of the burdát and aråbîr.

26 Erlanger [1949, v. 5, p. 11–12] explains briefly the evolution of the names for these degrees.

27 Degrees with orange background are the “neutral” e and b “half-flat”, and the degree n[g or g] determines the octave passage.

28 Or ÙUSHÅQ.

29 Or NÎRûZ.

30 The vowels of most of these names may be different because of the lack of the former in the copy; the names are cited on [Hîjazi (al-Malaki al-), 1864, p. 14].

31 The aråbîr figure on a dark green background (middle), the sīkâ and the nîrûz on light green-blue and green-orange backgrounds. The result is a scale divided in 28 conceptually equal “quarters” (column to the right).

32 We find clues to the latter and to the names used for the maqâmî in two previous works on Arabian music, the anonymous Aš-sh Shawkâr dâîr al-’Akmâm al-Hawwâya li-Uqîl al-Anghâm [Anonymous, 1983], and the treatise on music of a-s Şafa’dî ([Safadi (a.s.-)], 1991); Safâdî’s a-s-Safadi lived from 1296 to 1363; some Arabian scholars told me in verbal communications that this epistle may be falsely attributed to him; they did not produce, however, any proof for the latter, and I continue for the time being (until further information is retrieved on this epistle) to use the name of Safadi as the author of the Epîdâl in the science of music, albeit between square brackets in order to show that there may be an issue with the authorship. Detailed explanations about the two treatises cited can be found in [Beyhom, 2014] (forthcoming).
33 [Caron and Safvate, 1997, p. 26].
34 May they be “one-tone” or “three-quarter tones” intervals.
35 Let us note here that a certain Ibrâhîm Mustafâ seems to have developed a similar division of the octave, or argued about it, as writes Ghrab [2005, p. 71]: “Meanwhile, we have to notice the work of Ibrâhîm [sic] Bey Mustafâ [sic], [...] who contends that all bardât [the main intervals of the heptatonic scale] are divided into four parts to get 28 intervals by octave.” Ghrab cites as a reference for this author “the article of Ibrâhîm [sic] Bey Mustafâ, La valeur des intervalles dans la musique arabe [Value of intervals in the Arabic music], Bulletin de l'Institut Egyptien, II, 1889”. We could not find this article, but it would have surely been an interesting addition to the Egyptian point of view on Shihâb-al-Dîn's division of the octave.
36 In Erlanger’s descriptions of the Arabian general scale, transliterations of the names differ from ours and from other authors’ transliterations: this is a complicated matter as each European nation as well as various authors have used their own transliteration. Attempts have been made to unify the transliterations of Arabian terms, and two main systems co-exist today, one of which used in the Encyclopaedia of Islam while the other is used in the New Grove. I explain in my first volume on Arabic music (please see also note No. 19) the reasons why I think these transliterations should be adapted to fit more closely the pronunciation rather than the lettering of the Arabic terms.
37 [Erlanger, 1949, v. 5, p. 13 – Fig. 3]: reproduced by kind permission of the publisher.
38 For more details on this phenomenon see [Beyhom, 2014] (forthcoming).
40 See [Beyhom, 2005; 2006a; 2006b; 2007a; 2007b; 2007c; 2010c].
41 A few clues to this scale are given in [Beyhom, 2005], notably in the sections concerning figures 3.15 and 3.17 [p. 84 and 88], and figures 3.21 to 3.23. Figure 3.15 explains, notably, how the tuning of the ‘îd in fourths have probably affected the scale (or reciprocally), and how the one-tone-and-a-half intervals resulting from this tuning were probably divided in two equal parts (of the string or of the interval?) which led in turn to the zaqâliyah general scale. The resulting scale deduced from the A-sh-Shajara treatise is shown on Figure 3.17 [Beyhom, 2005, p. 88]. The base for this scale is proposed in the figure below.
42 Written probably around the 14th century or later (probably not later than the 17th century – see [Beyhom, 2014], forthcoming).
43 For this and all details concerning the A-sh-Shajara and the (a+) Šafâdî epistle, please consult [Beyhom, 2014] (forthcoming).
44 Other representations of the scale, mainly in the (a+) Šafâdî epistle, are possible and are shown explicitly in [Beyhom, 2014] (forthcoming). Two possible explanations of the “upper” and “lower” positioning of the ansâf are provided on the figures below.

The most probable position of the nusf of a bardât (degree of the Basic scale) is the upper one (this seems the case for all the “halves” cited in the A-sh-Shajara treatise, independently from the movement of the intervals, i.e. ascending or descending). Please note that bardât (pl. bardât) = “degree” or “interval”; nusf (pl. ansâf) = “half”; mutâqa = “free”; mutaqyad = “tied”; arîl = “higher, [ârîl min] higher than”; ansal = “low, lower, [ânsal min] lower than”.

The anonymous author uses expressions like (here for example for the mode Zînkûlā [Anonymous, 1983, p. 56] – please note that Zînkûlā (for the mode) is another transcription of ZÎNKULĀ (the degree of the scale)): “[then] you ascend [from the JAHRKĀ] to the half of the burdā of the BANJKĀ, or “تَصْدَعَ إِلَى نَصْفِ بَرْدَةِ البَانْجِكَةِ”. The following figure (below – which is an excerpt of [Beyhom, 2014], forthcoming) shows a possible conceptualisation of the A-sh-Shajara scale following diverse indications given in the treatise (including the numerous formulae in the treatise describing the “modes”). The main degrees (the bardât on the left, also called mutaqaqâd) are seven with each a corresponding nusf (“half”) to the right (the ansâf are also called mutqayyadâd); the total number of successive intervals between the bardât and the ansâf is 14. The main degrees of the scale are (ascending) RÂST → DUKÂ → SÎKÂ → JAHRKĀ → BANJKĀ → HUSAYNĪ → MAQLÛB [I use “AWJ” – the common modern name – instead in the figure] → Fawq A-r-RÂST. The circles shown on the figure are a possible interpretation of the phrase “the characteristics of the muqayyada and the mutaqaqâd are that for the mutaqaqâd the line going through the centre of the eyes arrives at the centre of the circle, and that for the muqayyada the said line arrives on the periphery”: 
in [Anonyme, 1983, p. 36].

This problematic was clarified in [Beyhom, 2007a; 2007e; 2010b]; a detailed discussion of the scales resulting from this interpretation are to be found in [Beyhom, 2014], forthcoming (see also endnote No. 47). The following are two excerpts giving an example of differentiation between upper and lower nuṣ (singular of ʿanṣāf).

Concerning the mode Ḥāṣāmānā: “[from the BANUKA] descending to the half of the burda of the JAHARQA, then to the SIRKA then to the DUKA and you rest on [it], then you avoid the SIRKA and you go up in one movement the upper half of the burda of the JAHARQA – [(Safadi (a-ṣ )), 1991, p. 152].”

Concerning the mode Nīrāt-Ārād: “descend [from the BANUKA] to the lower half of the burda of the JAHARQA – [(Safadi (a-ṣ )), 1991, p. 153].”

This division (below) is effectively one of the solutions for the explanations of the (a-ṣ) Safadi epistle on the composition of the modes – cf. [Beyhom, 2005; 2007a; 2007e; 2010b; 2014 – forthcoming] and the figure below (which is also an excerpt from [Beyhom, 2014], forthcoming).

Most notably (al-) Fārābī and (ibn) Sinā – please see [Beyhom, 2010c] for more details.

For other details on the scale construction of the Arabs from the 9th to the 13th century please see [Beyhom, 2010c].

To which a “disjunctive one-tone” interval is added in order to complete the octave. The “one-tone” interval was added in all possible three positions, before, between or after the two tetrachords.

The tetrachords are considered to be based on open strings, as with Rarral-Māya (13 3 4 3 4 4 in standard Modern quantification in quartertones) or Hijāgzūn (or Hijāz-Guarb) – 12 6 2 2 6 2 4 in Arabian music – see for example [Beyhom, 2003c, p. 56] and [Beyhom, 2010b, p. 34].

Most notably in [Beyhom, 2010c].

Other maqāmāt using this scale can be found in [Beyhom, 2003c, p. 57 – see hypersystem 4334334], of which an excerpt corresponding to the 43 3 4 3 3 4 scale is proposed below.

Also by ʿSāfiyya-a-d-Dīn al-Urmawī in his Risāla a-sh-Sahrūfiyya fī-n-Nisāb at-Taʿlīfiyya – cf. [Beyhom, 2007a; 2007e; 2010a; 2010b; 2014].

I follow here Owen Wright’s usage of the term zalzalian: “We shall term all species and scales containing neutral intervals Zalzalian, whether or not the wus Ẓalzal itself would have been used to produce them” – in [Wright, 1978, v. 28, p. 82, note No. 4]. The establishment of the zalzalian wusāt on the neck of the ʿud is explained for example in [Wright, 1978, v. 28, p. 31–32].

For example the division of half the string’s length in 24 quarters that Mashāqīq attributes to Sheikl al-ʿAttār in Damascus (see also note No. 17 on Mashāqīq above). This constitutes yet another riddle for which I give clues in [Beyhom, 2014].

See [Hassan et al., 2007], [Farmer and Chabrier, 2000] and, for a detailed discussion on the tambūr and the ʿud, Appendix A in [Beyhom, 2010c].

In what concerns the teaching and practice, please see note No. 60.

“The [nashār-kār] is a half-size Turkish ʿud with guitar pegs and six courses, used to be played mainly by amateurs; like other relics of Turkish influence, it has almost disappeared” – in [Hassan, 2001].

[Chrysanthos (de Madytos) and Pelopidēs, 1832, p. 28]. The title states: “The diatonic scale on the diapason system, on which the beginners are taught the quantity of intervals Zalzalian, which I give clues in [Beyhom, 2014].”

Chrysanthos explains: “Among the melodic instruments the one that appears easier for teaching and the one found to be the most clarifying for the learning of the tones, the semi tones and, simply, of all the intervals, is the pandoura. This is also called pāndoura and pāndouras and, by us, tamboura or tambour. It has two parts, the body and the neck. On the neck the tones and semitones can be fretted” – in [Chrysanthos (de Madytos) and Rōmanou, 1973, p. 24]. Chrysanthos further comments: “Among the melodic instruments the one that appears easier for teaching and the one found to be the most clarifying for the learning of the tones, the semi tones and, simply, of all the intervals, is the pandoura. This is also called pāndoura and pāndouras and, by us, tamboura or tambour. It has two parts, the body and the neck. On the neck the tones and semitones can be fretted” – in [Chrysanthos (de Madytos) and Rōmanou, 1973, p. 267]. “These names—in addition to tamboura and tambour—a designation in Byzantine and post-Byzantine periods the instrument known today as ‘bouzouki.’” (In Crete the name tamboura is still in use today.) The instrument has three pairs of strings at the intervals of a perfect fifth and a perfect fourth and is played with a plectrum. Its size varies from 0.70 m. to 1.00 m².

There is no evidence that Shihāb-a-d-Dīn worked with an instrument tuned with two strings a fifth apart, and we can not be sure
that such an instrument was used by him for his theoretical and practical researches on music (if any); we know however that these instruments were contemporary to the author and that other theoreticians of the same period, such as Chrysanthos (for Byzantine Chant) and Meshāqa (for Arabian music) explained their theories with the help of the ṭurūq. It would be most probable, anyhow, that a theoretician would use a lute-type instrument (with stoppable strings) for the establishment of his theory rather than his voice which would be to the least hesitant for the establishment of the degrees of the scale, especially for the intermediate notes between the main degrees; hints exist in the specialised Arabic literature concerning the use of instruments (in the following citation most probably a lute-type instrument) for a better location of the degrees of the scale, like this one in [Anonymous, 1983, p. 37]:

وأعلم أيضا أن تلك النصف الدرجة التي تتها على هي نصف نغة ومنها إلى نصف أخرى نغة كتابة ثالثا، وهكذا إلى جوانها، وإذ يakukan ذلك باللقب صب أو ينالها يعرف حقيقة قوله هذا، إنه بالتأكيد يمكن تهجينه وثلاثة بين بيئين، مثلن مقترحات.

"And know also that the ṭauṣ al-burdā [half of the interval a] we already cite is half of a ṭauṣma [melodic sound, note, degree, interval a] and from it to another half [you get] a complete ṭauṣma [you get to the next degree of the scale] then to the half of another a [then a] complete [second then] third ṭauṣma, and so on until the octave, [knowing that] it is difficult to perceive this with the voice [only] which makes it possible to know the truth of our present saying only through the use of [musical] instruments, because we can produce [with it] two or three notes between two distances, but they would be dissonant".

Beyond the difficulty of properly translating this passage, this is also a hint on the possible recourse to a division of the main intervals of the scale in three parts, as in the (a-) Ṣafādi treatise.

62 The Rāṣ scale with interval values (ascending) ↓4 3 3 4 3 3 would be found starting logically on c. Following this reasoning, the common Basic scale of the Arabs, as given in Figure 14, would then be a Ḥusnī type scale ↓3 4 4 3 4 3 (traditionally based on D) based on A.

63 See [Beyhom, 2010a, p. 177 – Fig. 5] and the next sections devoted to the śruti scale.

64 The G string is today often tuned in F in order to obtain a double octave from F on the first string till f on the sixth string.

65 Please note that both scales deduced from the A-ah-Shaqīrī treatise and from the (a-) Ṣafādi epistle may also be conceptualized as equal divisions of the string: a detailed treatment of this problematic is planned (as already mentioned) in [Beyhom, 2014] (forthcoming).

66 This instrument belongs to Saud Saab (Lebanon), who took the pictures for this and the next figure.

67 [Meshāqa, 1899a, plate inserted between p. 1076 & p. 1077].

68 Adapted from [Beyhom, 2010c, v. 1, p. 99 – Fig. 40]: this figure represents a stylised finger board of a common 'ūd; the vertical grid with fine blue lines shows the (approximate) positions of exact quarters of the tempered tone. The first mujannab (commonly known as "neutral tone" in Western specialised literature) resulting from this division (to the right) measures approx. 151 cents, and the second mujannab approx. 165 cents; "hd" stands for "half-flat" and " #" for "one comma plus" (with the comma (approximately equal to 24 cents) alteration). The "minor tone" between the ṭiṅs ("ring finger" or "annular") and the ṭiṅs ("little finger" or "auricular") positions measures 182 cents: the "major tones" in this construct lie between the perfect fourth and fifth. The fifth lower string was as a rule hypothetical; its first known appearance in practice is mentioned in the 11th century by (Ibn aṭ-) Ṭabḥān al-Mūsīqi (see the edition by Neuhauer [Ṭabḥān (Ibn aṭ- al-Mūsīqi), 1990, p. 177, f° 90r] or our transcription in [Beyhom, 2010c, v. 1, p. 504]:

"Concerning the strings: their number is originally four but some of the moderns tie a fifth string which they call the ṣār al-ḥākūt")

The Western notation of the resulting degrees of the scale (we use mainly the Arabian solmisation) is based on the Pythagorean interval basis, which shows the difficulties for the use of such a notation on the bīnār or waṣṭā line: in the Pythagorean system, flats and sharps lower or raise a note with the apotome (roughly equal to one limma + one comma, or 90c. + 24c. = 114c.). In the case of B, e, a and d (and the hypothetical g") on the figure, a more "traditional" Western notation would have been c (or e"), f, b and e; however, if the corresponding pitches c, f, b and e were to be placed (logically) at 408 cents from the nut (with a Pythagorean ratio of 64/81), c, f, b and e would correspond to their unaltered counterparts minus one apotome, (roughly) 408 – 114 = 294 c. This is however not the case, and the B, e, a and d notations reflect more faithfully the reality of the positioning, as adding an apotome to the B, e, a and d pitches from the Pythagorean position at 204 cents (with an 8/9 ratio) would give an interval of 204 + 114 = 318c. (to the cent) which is much closer to the 316c. value corresponding to the ratio 5/6. Please note that I have also avoided using "Just Intonation" notations in the figure as this would have been introducing a bias upon Arabian music, notwithstanding the fact that this would be anachronistic.

69 The upper string's perfect fifth interval (from the nut) is divided into 16 equal parts ("quarters") and the second's string perfect fourth interval (also from the nut) is divided into 12 equal parts. This is equivalent to the division of the octave in 28 "quarters", or equal parts of the string and quarts of the intervals between the 12 equal parts division, which gives as a result the main degrees of the Arabian scale.

70 [Jairazbhoy, 1975, p. 44].

71 Available in languages that I could read.

72 Like many other musicians or scholars, my first reaction to the śruti system was to consider them as equal intervals: "It has been necessary to clarify the connection between the seven swaras and Bharata's two series of śruti values before proceeding to the examination of the nature of the śruti themselves. Since Bharata distinguished twenty-two śruti within one octave, it seemed self-evident that one śruti was equal to one twenty-second of an octave; this would mean that the octave was divided into twenty-two equal parts. This was the generally accepted conception until the beginning of the 20th century," – in [Kolinski, 1961, p. 4].

73 I present in [Beyhom, 2014] (forthcoming) a few propositions for a better understanding of the latter theories.

74 Mainly his article on Indian music [Coomaraswamy, 1917].


76 And to the least beginning with the śruti scale found in Bharatamuni's treatise on music, as I further explain in the text.

77 A particularity of the Arabic language is that it has two different pluralis: the dual, and the plural proper; two mujannab(s) should be termed mujannabn, whenever more mujannab(s) (or mujannabn) would be transcribed mujannabn: as transcription of the Arabic language is already a complicated matter, we use the undifferentiated mujannab(s) for ease of understanding for the reader.

78 The range of thanks expands far beyond these few authors, but it would be too long to list them all here; please note also that a comprehensive and maybe helpful review on Indian music theories and their evolution can be found in [Powers and Widdess, 2001].
Alain Daniélou was the champion of such explanations, notably in [Daniélou, 1968], in which he uses [see for example p. 32-36] Pythagorean ratios to explain his “66-śruti scale”; the 66-śruti scale is also (and already) present in [Daniélou, 1949, p. 50-56], not to mention [Daniélou, 1943] and [Daniélou, 1959] in which Pythagoreanism is omnipresent; A. H. Fox Strangways openly acknowledges in [Strangways, 1908, p. 30] that his “article seeks to establish some underlying principles for Hindu rāgas, to trace the connection between the early music of Greece and of India”, and deploys considerable efforts in his book on “Hindustani music” in order to express the sizes of the intervals in the scale in Pythagorean-like ratios. Despite his frequent references to Aristoxenus (see for example [Strangways, 1908, p. 464] and [1914, p. 103, 114, 125, 156 etc.]), the author’s scale in his table [Strangways, 1965, p. 117 of the lithographic reprint of the 1914 edition] contains three different śruti the sizes of which 22, 70 and 90 cents; the final construct carefully avoids any “neutral” tones in the scale. However, “[the crucial question […] is whether the system as a whole is based on the cyclic or on the divisive principle, to use the terms suggested by Sachs,” (as put in [Kolinski, 1961, p. 4]), knowing that “[the] divisive hypothesis appears more likely [that] 7 śruti represented the major third 4:5 or 8:10, and that this interval has been divided into the major whole tone 8:9 of 4 śruti and the minor whole tone 9:10 of 3 śruti.” – in [Kolinski, 1961, p. 5].

“The number of the śruti in the Saṣād Grāma are as follows: three in Raṣṭha (ṛ), four in Gāndhāra (ga), four in Madhyama (ma), four in Parivara (pa), three in Dhaivata (ṣ), two in the Nādī (n) and four in the Saṣād (sa)” – [Bharata, 1961, v. 1581, p. 6 (XXVIII.25.26 & XXVIII.27.28)]; this scale corresponds to ↑3 3 4 3 4 in quartertones, with a bayāt + rāst ascending tetrachordal structure – or bayāt ↑3 3 4 + disjunctive tone 4 + bayāt ↑3 3 4 in Arabic music. By beginning on sa instead of ra, we obtain the ↓4 3 2 4 3 2 (in śruti) scale. The author further states (same page) that the structure of the Madhyama Grāma is as follows: ↓4 3 2 4 3 2 (in śruti and beginning with ma), which is a perfect conceptual match for the scale of maqām Rāṣ-h-a-d-Dīl as shown in the figure below (taken from [Beyhom, 2003b, p. 56]), and for a few others maqām(s) (different names in different maqām regions).

Rāṣ-h-a-d-Dīl Ri 0.195.3.4344343

- I.e. a scale using the so-called “neutral tones”: the word “zaļatian” originates in the name of Mansūr Zaļaz, an 8th to 9th century ’id player at the ’Abbāsid court of Baghdad, reputed to be the first to use positions for “neutral thirds” on the fretboard (see for example [Farmer, 2001]). The question whether “zaļatian” (or “zaļataly”) originated with this musician remains however highly controversial.

- In his book *The modal system of Arab and Persian music: A.D. 1250-1300* Wright explains how the Pythagorean positionings of the pitches in Urmawi’s theory are to be considered zaļatian, i.e. based on intervals approximately equal to the 3-quarter or 5-quartertones used in Modern Arabian theories of the scale. Saḥfy-a-d-Dīn al-Urmawi even formulated in his second epistle on music (the Rūdā Shākhāyīya – see [Urmawi (al-), 1982; 1983]) an explicit zaļatian third (which he calls “Persian” for reasons explicit in [Wright, 1978]) and that I further explain in [Beyhom, 2010b] and in [Beyhom, 2014] – forthcoming) with the ratio 59/72 corresponding to 345 c. and an explicit zaļatian second with the ratio 59/64 corresponding to 141 c., as I have already reminded in a number of papers ([Beyhom, 2006a; 2007a; 2007c]). Both these zaļatian third and second are found on the neck by halving the string length corresponding to other intervals obtained through a Pythagorean construct, which shows that the equal division of the string is one of the ways used by the Arabs in order to include zaļatian intervals in a scale (see for example [Urmawi (al-) and Jurjānī (al-), 2001, v. 3, p. 110–120], with also useful information on the undifferentiated use of the ṭyannābdīt for describing the tetrachords of Arabian music by Urmawi). I have also explained (in [Beyhom, 2010a]) how Saḥfy-a-d-Dīn’s “Pythagorean” theory is, basically, an attempt to force zaļatian intervals in a Pythagorean costume – more details on Urmawi’s zaļatian conception of the scale are to be found in [Beyhom, 2014] (forthcoming).

- For example, a 3 śruti interval is always greater than a 2 śruti interval in the course of, for example once again, one same melodic phrase. Proportionality is the rule, but the exact measures of the intervals may (and do) vary.

- “The Viśnī is tuned thus; the two lowest strings at the distance of a Fifth, the rest in Fourths. It has at present four fretted strings on the fingerboard and three unfretted at the side played as a drone by the (armed) little finger of the right hand.” – in [Strangways, 1908, p. 454], and: “There is little doubt that the consonance of fourths and fifths was of fundamental importance in ancient Indian music” – in [Jairazbhoy, 1975, p. 42].

- (Ref. 6; p. 114) Here, Kolinski refers to [Strangways, 1914, p. 114]: “It appears from the table of mārchaṇs that all the twenty-two śruti except the first and twenty-first are accounted for. These two are inserted, by analogy, in the next diagram in square brackets as consonant notes from the eighth and tenth śruti respectively.”

- Here, Kolinski refers to [Daniélou, 1943, p. 121–122], in which the latter notates (p. 122): “If we exclude from this series G + (Pa +) (Ab), the fifth being invariable, we obtain a scale of twenty-two sounds, the śruti”; Daniélou furthers compares [1943, p. 122–123] the 22-śruti system to the “Arabian” and “Ancient Greek” scales: “This scale is identical to the one given by Arab mathematicians as having been that of the ancient Greeks, and it still remains the division used by the Arabs themselves. The major tone is thus divided into minor tone, apotome (or major half-tone) and limma”; this shows that Daniélou can hardly be considered as a supporter of the “divisive” theory, but should rather be considered as a promoter of the Pythagorean (“cyclic”) system applied to the Indian scale.

- Kolinski refers here to [Clements, 1913, p. 101], in which the author concludes: “The fallacy underlying the theory of the equality of the śruti is demonstrated by the numbers given. They are calculated on the basis that a one-śruti interval is 22 cents, two śruti 112, three śruti 182, and four śruti 204. The 3 śruti interval of the Gandhrava Grāma is 134 as explained in the text. It will be seen that the ancient system required 25 śruti, and not 22, three of them being confounded with their neighbors”.

- [Kolinski, 1961, p. 5]; Powers, in his review of Kolinski’s article [1962], strongly criticises some major points of his reasoning and claims ([Powers, 1962, p. 223]) that “Mr. Kolinski’s basic premise is that the system of 22 śruti must somehow or other be connected with the ‘cyclic’ method of tuning by fifths, rather than with the ‘divisive’ method based on just intonation”. Please note that the only systematic homogenization of the transference of Indian musical terms (throughout the numerous citations in the article) was applied to the word śruti. Most of the other transferences were left unchanged in order to reflect the time, but also the place of transliteration.

- [Coomaraswamy, 1917, p. 165];

- [Kolinski, 1961, p. 4-5].

- “[S]ince each of the twenty-two śruti has its proper name, one should infer that each of these twenty-two names has its distinctive meaning. Does this, then, involve the assumption of a basic division of the octave into twenty-two tones? By no means. Both the general tonal
structure of Indian music and the nature of the quintal principle from which, as we saw, the 22-śruti complex must have originated converge into the concept of a system of twelve tones within the octave, that is, a collective chromatic scale into which all heptatonic and other Indian scales and modes may be projected if differentiations in intonation of secondary structural importance are not taken into account” – in [Kolinski, 1961, p. 6].

92 [Coomaraswamy, 1930].

93 As Jairazbhoy [Jairazbhoy, 1975, p. 54 – note nº 12] puts it: “Coomaraswamy (referring to [Coomaraswamy, 1930]), has argued, on the strength of textual descriptions, that the ancient Indian vina was a bow harp. This seems to be corroborated by early Indian bas-reliefs. Bake (referring to [Bake, 1957]) has, on the other hand, argued with some justification that Bharata’s experiment only makes musical sense on a stopped stringed instrument”. Another corollary assumption is that the śruti is not necessarily an equal size interval over the octave, as Popley [1921, p. 26] puts it: “The śruti or microtonal interval is a division of the semitone, but not necessarily an equal division” (although I do not really understand why the śruti should divide the semitone, and not the tone or the fourth as two other possible examples) – see also [Dick, Wickless, and Goksis, 2001], notably: "In South Asia, short-necked lutes first appear in the Graeco-Buddhist art to the end of the millennium. They generally occur in the same contexts as harps”. Let us note that the equality of the śrūtis may also have been a hypothesis of Indian musicologists and researchers or musicians: “Um die europäischen Forscher mit der genauen Intervallgröße der Śrūtas bekannt zu machen, sandte Tagore 1886 an Ellis eine Vīṇā, auf der die vollständige 22-stufige Leiter durch feste Bünde fixiert war. Die Teilung der Oktave war in der Weise vorgenommen, daß die Seiten-länge in zwei Hälften, die so entstehende untere Quarte in 9, die obere Quinte in 13 gleiche Teile zerlegt wurde. Ellis bestimmte die den Bünden der Śrūta-Vīṇā entsprechenden Tonhöhen und berechnete, vom Grundton aus folgende Werte in Cents (Hunderstel des temperierten Halbtons): 0 45 111 169 222 267 316 389 436 505 534 583 640 712 […] 749 807 855 917 954 1013 1077 1136 1220 […] Ellis vermutet, daβ eine 22-stufige temperierte Leiter intendiert war” – in [Abraham and Hornibostel, 1904, p. 382].

94 To whom I am indebted for most of my practical knowledge on Middle-Eastern Arabian music today.

95 The different kinds of instruments, with different tunings.

96 In traditional non-standardised instrument making, the “same” two instruments can have differences, albeit sometimes small, in tunings, measurements etc.

97 On the fingerboard of a ‘ūd, for example, thicker finger tips or smaller hands (or longer fingers) can change the way in which the musician performs, thus inserting additional (sometimes very small) discrepancies of intonation between the intervals used by two different musicians; such differentiations of intonation remain whatever the musical practice is, as long as the instruments themselves are not completely standardised and equally tempered. This, and other factors which contribute in introducing differences of intonation and heterophony into modal music are discussed in some of my writings, including [Beyhom, 2001; 2003a; 2003b; 2003d; 2004; 2007c; 2008; 2010a], and especially [Beyhom, 2007d].

98 Or for its transposed equivalents.

99 This is common knowledge for any educated musician or teacher (of Arabian traditional music) in the Middle-East.

100 The term “natural” should be considered with considerable caution here: this expression is, to the least in this article and in my other writings, used in a mere conventional way in order to indicate that the degree e conforms to the usual unaltered e in the Western scale.

101 (Reminder): Plural of mujannab, a term used in Ancient Arabian manuscripts to define the position of the finger, on the fingerboard of the ‘ūd, for what was to be called “neutral tones” (i.e. tones that are neither “major” nor “minor” in Western music standard theory) by Western musicologists.

102 In the Kitāb al-Adwār – see one of the references [Urmawī (al-), 1980; 1984; 1986; 1938; 2001], and [Beyhom, 2010a].

103 Information about performance practice in Urmawī’s writings is very scarce: all details on this subject are to be found in Owen Wright’s magisterial book on the Systematists [Wright, 1978].

104 More detailed information about Urmawī’s use of the mujannabāt is to be found in [Beyhom, 2010a], and in [Beyhom, 2014] (forthcoming).

105 This is the replica of Figure 5 in [Beyhom, 2010a].

106 [Urmawī (al-), 2001, p. 6].

107 Please note that in Urmawī’s theory two consecutive mujannab(s) are never equal and have the form (L C + L I) or (L I + L C), the total of which is a “minor” third equal to one tone plus one limma (or 3L + C, as the tone value is L I, or two limma plus one comma, in combination). The reader may find detailed explanations on the different type of intervals used in Arabian music theories beginning with the 9th century and on the way they are used in these theories, especially in Urmawī’s Book of cycles.

108 For ancient Arabian music theories, see [Beyhom, 2010c]; for modern performance, this is a reality of today’s teaching and of yesterday’s (the turn of the 19th to the 20th century) music on old records (see [Beyhom, 2014], forthcoming).

109 Indian music specialists compare sometimes the “3 śrūtis” interval to a “small”, or “minor”, tone, and the “2 śrūtis” one to a “semitone” – see for example [Popley, 1921, p. 31], or [Bake, 1957, p. 61]: “Indian music recognizes two, three, and four-śrūti tones which roughly correspond with our semi, minor, and major tones”. The same author asserts: “As it was quite clear, even after the first attempt to translate Bharata’s extremely concise text, that this premāṇa-śruti was an interval equal to the difference between a major and a minor tone, investigators accustomed to the mathematical approach of the Greeks to their music, at once applied Greek standards to determine the measurement of the standard śruti (comma of Didymus) and from those premises began detailed calculations as to the exact measurement of the 22 śrūtis which find a place within the compass of the Indian octave” – [Bake, 1957, ibid.].

110 [Kolinski, 1961, p. 3].

111 See [Beyhom, 2010c, Appendix A].

112 To the least in Arabian music, contemporary and Ancient: the (somewhat successful) attempts to depict ancient ‘ūd’s as “fretted” are mere attempts to impose a fixed temperament (often based on a Pythagorean division of the octave) to Ancient Arabian music – see [Beyhom, 2010c, Appendix A] and [Beyhom and Makhlouf, 2009], as well as [Beyhom, 2011].

113 Especially when these positions are determined by complex ratios such as the ones used for the Pythagorean limma and comma, for example.

114 This is an exact copy from [Subramanian, 1985, p. 12 – Fig. 8] previously used for the exposé on the origins of the ‘ūd in [Beyhom, 2010c, v. 1, p. 304 – Fig. 105].

115 From [Marcel-Dubois, 1937, Fig. 1]: by kind permission of Rosy Azar Beyhom who made the line drawing.
116 Please note that there is no origin issue here as the Ancient Indian treatises predate with no doubt the first Islamic treatises on music, as well as the vīnā prefaces (to our knowledge) the ādī – see for example [Jairazbhoy, 1972, p. 63]: “Musical theory in India stems from the Nāyāsastra, ascribed to the author Bharata, which is generally dated from the second to the fifth century A.D.”. As the first extant writings on Arabian music theory are the epistles of (al-) Kāndī, the Philosopher of the Arabs (9th century), maqāna is only used conveniently as an interval which is well known in Arabian music theories.

117 Logarithmic computation is relatively modern though musicologists tend to forget about it. As a consequence many believe that “equality” can only be conceived in modern terms.

118 These numbers are rounded to the closest integer unit. Please note that, obviously, multiplying 55 cents by 22 śrūti will give us a value which is not a perfect match for the octave (exactly 1217.44 if using the accurate mean value of the śrūti – rounded to 1217 cents to an octave, which is 17 cents surplus); this is however no issue for an Ancient theorist because the octave is still divided in 22 equal śrūti, the ones used to divide the perfect fourth in 9.

119 “The two main theories which find support are both based on sound musicological principles. The first of these, described by Fox Strangways, derives the śrūti from the ‘divisive’ principle where the tones are determined on the basis of simple fractions of string length. The second, described by Kolinsky, derives the śrūti by the ‘cyclic’ or ‘up and down’ method in which the tones are determined by perfect fourths and fifths. […] Both theories arrive at the conclusion that the śrūtis were of three different sizes; 22, 70 and 90 cents in the ‘divisive’ and 24, 66 and 90 cents in the ‘cyclic’. The evidence in the Nāyāsastra, however, seems to suggest that the śrūtis were of one constant size, or at least, that they were thought to be so.” – in [Jairazbhoy, 1975, p. 38].

120 Except for transpositions to the perfect fourth, due to the nature of the tuning and of the division (the first in perfect fourth and the second dividing this interval in equal parts). This can be easily checked on Figure 20.

121 It could however bear parallel marks indicating the theoretical positions of the śrūti division, or other small marks playing the same role on the top of it.

122 As already explained above in the text.

123 The octave interval is not, for example, a necessary characteristic of the maqām scale, as some maqāms(s), and specifically maqām Šāhī of Arabian music for example, are constructed in such a way that they avoid the octave interval (in this case the ascending scale as can be seen on the figure below from [Erlanger, 1949, v. 5, p. 282 – Fig. 123], reproduced by kind permission of the publisher).

124 Let’s also remember that the octave is not an interval resulting from the cycle of fifths cherished by most musicologists dealing with Indian music – see [Beyhom, 2010a; 2010c, v. 1, p. 56–70].

125 Jairazbhoy’s explanation on this subject ([Jairazbhoy, 1975, p. 54]) is noteworthy: “The total number of śrūtis in the octave, twenty-two, is only incidental, being determined by the size of the unit of measure”.

126 In [Beyhom, 2010a] as one example.

127 The same does not apply to the ratio between Mₚ and T expressed in cents and expressed in śrūti, as 2/3 (=0.67) and 3/4 (0.75) are nearly the same.

128 In the “3 2 1 3 2 1 3” division, the fourth’s value would be 3+2+1+6 elementary intervals (śrūtis?), which means that the string’s division is on the basis of 24 division in all (the perfect fourth emplacement on the neck is at one fourth of the string and it contains 6 elementary intervals – this corresponds to a division of the string in 24 equal string-parts, and of the octave – from the nut to half of the string – in 12-equal-string-parts).

129 As (24-3)/24=21/24=7/8. If we were to divide the half of the string in 15 (which is the sum of the elementary intervals in the “3 2 1 3 2 1 3” division), the ratio would be based on a division of the string in 30 equal-parts (15 for the octave between the nut and the half of the string, and 15 for the other half of the string), and the ratio of the first 3 elementary intervals (the “tone”) would be 27/30, or 9/10. In other terms, the first result (7/8) is based on a tuning of the strings in fourths and the subsequent division of the length of the fourths in 6 equal string-parts, whenever the second result (9/10) is based on a division of half of the string in the 15 elementary intervals that would result as a whole from the “3 2 1 3 2 1 3” division. This is just another example of the numerous possible uses of the equal division of the string technique.

130 It is much easier to get to the 4/5 ratio than to the 29/36 ratio as the calculations are much simpler (and an eventual folding of the string even simpler); if it was to be used as such in the division of the string, however, this would have changed all the overall division in equal string-parts which I think is the basis of the śrūts system.

131 Compare this discussion with: “Perhaps a musicologist could have determined empirically that tones of three different sizes were used in Samavedic chant. Further, he may have determined that the largest tone was about double the size of the smallest and the third tone was somewhere between these two in size. Since presumably he had no way of determining the size of this intermediate tone with any accuracy, nor an objective standard of intervallic measure against which to compare it, the obvious way would be to attempt to relate it to the other tones. In practice this is virtually impossible to do by ear alone and the most convenient approximation which suggests itself is to consider it as being half-way. Thus, if the smallest tone is expressed by the numeral one, the large tone would be two and the intermediate tone one and a half. Fractions are clumsy to handle and in this case would easily be eliminated by doubling each of these numbers. This would mean that the size of the small tone is now assigned the number, two; the intermediate tone, three; and the large tone, four. These are in fact, the śrūti values of the tones given in the Nāyāsastra” – [Jairazbhoy, 1975, p. 52]. Please note also that number 36 can be divided by a variety of smaller numbers like (1) 2, 3, 4, 6, 9, 12 and 16; this gives many possibilities for ratio simplifications, as we can see for example for the second, third and fourth śrūti on Figure 20 (with ratios 17/18, 11/12, 8/9, as well as the sixth, the eighth and the ninth (with ratios 5/6, 7/9 and 3/4).

132 As opposed to “quantitative”.

133 With probably an exception in what concerns the perfect fourths and fifths; this is the case in most of the theories of modality, including maqām and Byzantine chant theories, as we show in [Beyhom, 2014] and [Beyhom, 2013] (forthcoming).

134 For example applying to this scheme the well-known experience of the two vīnās of Bharata Muni [Bharata, 1961, v. 1581, p. 7–9]: due to the impossibility to check by myself the original manuscripts (and language – these two conditions are, in my experience, very important because of the tendency of the commentators to interpret the manuscripts at their convenience), I simply can not know if this experience is compatible with the equal string-division of the fourth.

135 It seems however that the 28-quarters division of Shihāb-ud-Dīn is superfluous as a conceptual construct, as only 14 or 17 degrees in
the general scale have names of their own; the further division in Safinat al-Mulk of the 14 ansāf (or halves) found in previous theories seems to be a step towards more accuracy in the determination of interval sizes or degree positions, i.e. supplementary intermediate positions between the degrees used for small intonations or unusual transpositions. On the other hand, the fact that almost all the degrees (or the intermediate intervals between them) of the old Indian scale are accounted for (see Strangways, 1914, p. 114) seems to mean that the 22-śruti construct is conceptual in its essence (see Beyhom, 2010a) for more details about conceptual and measuring theories and their differentiation), or “more” conceptual.

Apart from the tempered instruments of modern Europe there scarcely exists an absolutely fixed scale. [...] [T]he meaning of the śruti concept has to be discussed. Was it but a simple expedient to determine roughly the three different sizes of the svaras or did it involve an actual subdivision of the octave into twenty-two tones? The way in which Bharata utilizes the śruti hints at the former interpretation” – in Coomaraswamy, 1917, p. 165).

Or aiming at fixing.

The measurement of string and pipe lengths was conceivable since earliest times.

See for example [Jairazbhoy, 2008] concerning this point.

The b and e degrees could be here considered as zalzalian (and noted bhf and ehf) if not for the discrepancy between the corresponding interval and its “standardised” zalzalian form (on Figure 20: 350 c. from the nut).