MODUS VIVENDI1

Richard Dumbrill*

Foreword

It is not possible, objectively, to define the original concept of mode since what we think it is may no longer be what it had been, and while we may satisfy ourselves with our current perception of it, a reconsideration of its origins needs to be addressed for the sake of academic probity. Every assumed exogenous and modern instance of it, is no longer the reflection of its possible historical and, or, ethnical authenticity for it has been contaminated by western dictates, for the past millennium, if not longer. And any instance of its assumed endogenous occurrence has also been corrupted in the course of the past two millennia, or more, by political, philosophical or theological² ideologies to suit whomever. Therefore, however futile may seem to be the purpose of this disquisition, it is nevertheless essential as a record of what we think might remain of it at the dawn of the twenty first century, and therefore is an academically defensible exercise.

INTRODUCTION

Both "tone" and "mode" are inappropriate terms of imprecise meanings which are used to describe ill-defined pitches or pitch sequences, quantitatively and qualitatively. Greek and Latin dictionaries agree that "tone" stems from the Greek $\tau \acute{o}vo\varsigma$.

Tóvoi were modes or keys differing in pitch. The Latin "tonus" is the sound, tone, of an instrument, and the term is therefore slightly more accurate. But Latin "modus" is the measure of tones, melody, rhythm, and time³. Thus "tone" and "mode" appear substitutable. "Mode" is also anachronic since in Early Greek contexts, but in modern argumentation, it defines something which had not yet been known, as a term, and probably not as the concept with which it is usually associated, "a priori". Furthermore, it is still of common belief, even at the dawn

of the 21st century, that these terms may apply, erroneously, to all known systems⁴, intemporally and interculturally, obscurum per obscurius. This attitude may be construed either as musical neo-imperialism where all is ruled, measured, codified, notated, compared, studied and published by western scholarship. exclusively, and is strongly conditioned by Hellenocentric a prioricity since this position finds reasonable to infer anything without any empirical evidence, infallibly, because in this case nothing can be taken as evidence against it.

There is a profusion of respectable reference volumes giving corpulent definitions which for the most obnubilate rather than enlighten. Classical metrology of musical systems is nothing but subjectivist convention and is therefore inappropriate. Additionally, Greek roots and etymology, although very convenient, tend to relate all that we qualify with them, to Ancient Greece.

Then, in the West, Roman; Mediaeval; Renaissance; Classical; Romantic and modern treatises have in the course of the centuries added to the confusion brought up by an almost universal belief in the reliability of Greek knowledge transmission. There are no Classical autographs. We have only late copies dating 1000 to 1500 years after their assumed composition, and mostly dating around the 11th century a.d. and later6, and Classical philologists may argue that on the basis of "x", "y", or "z", that what we have from early to late Greek theoreticians was indeed from their own hands. However, without autographic material, I remain cautious. Mediaeval theoreticians often used the Greek medium for writing their own treatises making it difficult to distinguish their works from copies of older material. The meagre fragments of musical theory extracted from Oxyrhynchus, for instance, are hardly evidential of Aristoxenus' work; and Suidas' 10th century Byzantine Suda is more of a biographical index than it is a collation of theses, and is therefore of little academic value for our purposes.8

Consequently, my attempt at defining the elusiveness of what mode may have been be will be within archaeological sources of music theory, philology and iconography. The evidential material, should mode had ever been intended to be transcribed, may constitute its earliest appearance insofar as the cuneiform texts originating from the Ancient Near East are autographic, for the most, and date from about 2200 B.C., to around 600 B.C., at the dawn of Greek Orientalism, covering a period of some 1600 years of music theory. This is a considerable period which has been researched, for the past fifty years, mainly by philologists of the musicological background and the results are consequently, for the period under disquisition, of little scientific value.

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THE EVIDENCE

The documentary evidence appears in the form of clay tablets written in the cuneiform system of writing. Some are written with agglutinative¹¹ non Semitic languages such as Sumerian and Hurrian, and others with Semitic languages¹² such as Akkadian and its development into Old, Middle and Neo-Babylonian and Old, Middle and Neo-Assyrian, for the Semitic. There are bilingual lexical texts written in Sumerian and Akkadian. These texts are invaluable for each other's elucidation. Some texts have been unearthed in context and are therefore datable with great precision but other documents, mostly having been acquired in the 19th century from various antiquarians can neither be sourced nor dated with certainty¹³. However, the nature of their content and sometimes their orthography may become useful for refining their identification.

The texts are of three types. Firstly there are mathematical documents which are essential in determining the counting system used for assumed pitch quantification, as the inhabitants of ancient Mesopotamia had different systems for different purposes, mainly sexagesimal (base 60) and decimal (base 10). Secondly, there is a lexical and literary texts. Thirdly we have theoretical texts and fourthly there is musical notation of songs, in Hurrian, constituting the "proof in the pudding", as if the theory is well interpreted, then the written instructions will yield music lending itself to analysis and not amount to an aleatory succession of pitches which might have come straight from a "laundry list". 14 Often, seal cylinders and other forms of iconography may contribute to the elucidation of the philological and theoretical narratives, and in relation to glyptographic contribution.

Most of the cuneiform texts we shall investigate in the present paper are autographic. Some are copies of older texts. However, scribal reliability is recognised in the Ancient Near East. ¹⁵ Thus we are quite confident with the quality of the transmission of the cuneiform corpus which is otherwise proven in the consistency of the terminology and its orthography often spreading through two millennia, and more, of scholarship.

THE TEXTS:

Mathematical texts: CBM 11340 + 11402, obv. and rev.; CBM 11368, rev.; CBM 11902, obv.; CBM 11097, rev. These four texts were unearthed at the site of the Temple Library of Nippur in Southern Iraq, during the Babylonian Expedition of the University of Pennsylvania. Out of fifty thousand tablets found at the site, seven thousand of them were catalogued and published by Hilprecht in 1906.¹⁶ More

- recently, Margaux Bousquet¹⁷ and Leon Crickmore¹⁸ have re-evaluated other texts.
- Lexical text: nabnītu¹⁹ XXXII (UET VII, 126 = U.3011), obv., cols. i and ii. This text gives the names, number and qualifiers of a series of nine strings in both Sumerian and Akkadian. It was unearthed at Ur, at the site of Dublamah, south of the main courtyard in the late twenties by Sir Leonard Woolley²⁰ who gave it the field number U.3011. It is a late Babylonian copy of 32nd tablet of the series *nabnītu*, one of the great lexical texts. It was originally published by Kilmer²¹ in one of her early papers and published again by her in 1965.22 The late Professor Gurney of the University of Oxford published his hand copy of the tablet in his VIIth volume of the Ur Excavation Texts²³ (UET) and renamed it UET VII, 126, being the 126th text in his volume. Although this is a late copy of the first millennium B.C., I will advance that the Sumerian nomenclature originally dates from the early to mid-third millennium B.C.
- 3. Text of theory. *CBS* 10996. This tablet was published by Kilmer in 1960²⁴. It was found at the site of Nippur and was originally thought to be from the Kassite Period, about 1500 B.C. It is probably Neo-Babylonian, early first millennium B.C. It lists a series of intervals "adapted" to a heptachordal instrument. However, the evidence and extrapolation reveal that the text had been devised, originally, for a span of 13 degrees. On this basis, the original theory would have dated from the early to mid-third millennium B.C. since the few stringed instruments in the iconography with a large amount of strings date from that period.
- 4. Text of Theory. UET VII, 74. This cuneiform tablet dates from the Old-Babylonian Period, about 1800-1750 B.C. It was excavated by Sir Leonard Woolley at Ur in southern Iraq and was published by Gurney²⁵ in 1968, and by others. This text has generally been mislabelled as a "tuning text" and a "re-tuning text". It is neither for the reason that it does not say how to tune anything. It gives instructions for the construction of a system stemming from a generative pitch set the tuning method of which not being provided.
- CBS 1766. Text of theory. This tablet with a heptagram inscribed in concentric circles shows evidence of an unqualified

heptatonic tuning system along with a possible device, a "computer" to guide the musician in his tuning and scale construction. It is unprovenanced and undated because it was not found in context but was acquired from the Khabaza collection²⁶. The tablet is hosted at the University Museum of the University of Pennsylvania. However, many of the tablets in the collection seem to originate from Nippur. On the basis of its contents, I would date it from 1200 to 800 B.C.

6. Music notation. H. 6 = (RŠ 13.30 + 15.49 + 17.387). The tablet I have chosen to illustrate my argumentation is the only one, out of 29, which could be fully reconstructed from 3 fragments. It was excavated during pre and post Second World War French missions at Ras Shamra, (Ugarit) Northwest Syria, conducted by the French scholar Claude Schaeffer.²⁷

I - Mathematical Texts

CBM 11340 + 11402, obv. and rev.; *CBM* 11368, rev.; *CBM* 11902, obv.; *CBM* 11097, rev.

Sometime in 2007 I was researching cuneiform mathematical texts which might be inscribed with pitch quantification. More precisely, I was looking for numbers giving ratios of Just Intonation between them. Ancient Near Eastern music theoreticians would have used the sexagesimal system rather than the decimal, as the former is ideally suited to Just Intonation²⁸. My investigation led me to Hilprecht's work where I found what I was looking for²⁹. Margaux Bousquet's and Leon Crickmore's aforementioned work confirm that these tables were well recognised, from pre-Hammurabi Elam, to Nippur and Sippar, and date from around 2200 B.C.

Hilprecht referred to the texts as tables of multiplication and division. However, he did not fully understand their purpose because in 1906, when he published them, texts of musical theory, which would have focused his mind on musicology, had not yet been published ³⁰. In all cases, Elamite and Babylonian, the tablets share two principal features:

- The numbers inscribed are not consecutive. They
 are often separated from each other by
 comparatively large intervals.
- 2. Besides 3 and 5, no indivisible number or its multiple is multiplied and therefore there is absence of 7; 11; 13; 14; 17; 19; 21; 22; 23; 26; 28; 29; 31; 33; 34; 35; 37; 38; 39; 41; 42; 43; 44; 46; 47; 49; 51; 52; 53; 55; 56; 57; 58; 59; 61; 62; 63; 65; 66; 67; 68; 69; 70; 71; 73; 74; 75; 76; 77; 78 and 79.

The remaining numbers are regular numbers³¹ as they evenly divide powers of 60. They can be characterized as having only 2, 3, or 5 as prime factors. This is a specific case of the more general k-smooth numbers, i.e., a set of numbers that have no prime factor greater than k. In music theory, regular numbers occur in the ratios of tones of Just Intonation, also called "5-limit tuning" for this reason. Thus all remaining numbers would quantify a descending diatonic pitch set of Just Intonation from 27 to 81, descending because the ratios arising from them would be ratios of string lengths rather than ratios of frequency, and composed of diatonic intervals of just intonation d-c-b-a-g-f-e-d-c-b-a-g, on the basis of the ratios formed by the quantifiers which are regular numbers (2^p3^q5^r). While it would be unreasonable to assume that these tables were only used for the purpose of music theory, it would be equally unreasonable to assume that they were not used for it.

The four fragmentary texts can be reconstructed as one table as follows:

1	$8.640.000 A-AN^{24}$	25	518.000
2	6.480.000	27	480.000
3	4.320.000	30	432.000
4	3.240.000	32	405.000
5	2.592.000	36	360.000
6	2.160.000	40	324.000
8	1.620.000	45	288.000
9	1.440.000	48	270.000
10	1.296.000	50	259.000
12	1.080.000	54	240.000
15	864.000	60	216.000
16	810.000	64	202.500
18	720.000	72	180.000
20	648.000	80	162.000
24	540.000	81	160.000

Fig. 1. Reconstructed table³².

Hilprecht attempted at finding a reason for this table and found some answers in Plato's Republic, Book VIII, 546, B-D.33 It makes little doubt that Plato attempted at what proved to be a very successful numerologicalmythological manipulation of a much older Babylonian story to which he never referred, as far as we know. As basis for all of his calculations, he uses the Pythagorean triangle. The right-angled triangle in question is one in which both sides are 3 and 4 with a hypotenuse of 5, naturally.34 The right-angled triangle has sides which measure 3, 4 and 5. Therefore they have 3:4:5 as ratios between them. The ratio of 5:6 is made up from the doubling of side 3 in relation to the hypotenuse. Ratios of 1:2 and 2:3 arise from the halving of 4. Thus we have 1:2; 2:3; 3:4; 4:5 and 5:6. These ratios correspond to the first divisors in Hilprecht's reconstruction. However, the divisor "1" should relate to 12,960,000, and not to 8,640,000 whose divisor should be 11/2. Hilprecht was concerned by this discrepancy (while another problem was discussed by Scheil³⁵) and writes:

"I am unable to explain this strange phenomenon. Possibly we have to regard it as an abbreviated expression well understood by the Babylonians". 36

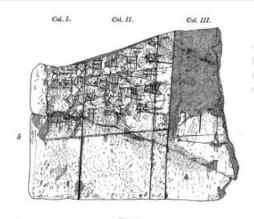
I do not see, either, any reason for this other than an irrational one, or, as Leon Crickmore puts it to me, in a private communication:

"... could line one, for example, be a concession to practical musicians, who are not generally noted for their mathematical expertise? Or, could it be a reminder for theoretical musicians that the whole of these tables can have an application in a musical context? Or is it simply the scribe's dedication of the table to Ea, the 'god' of music?". ³⁷

Indeed, if we read the sign as $\check{su}\check{su}=60$, god Anu's number, referring to the musical string of 60 $uban\bar{a}tu$ (fingers), 38 then 60 x 2/3 = 40 which is god Ea's number 39 . The table which follows gives the full range of regular numbers, their ratios and corresponding pitches transcribed from our mathematical tablets.

Number	Pitch	Ratio	Number	Pitch	Ratio
2	B ⁷		27	D ⁴	
		3:2			10:9
3	E7		30	C ⁴	1
		4:3			16:15
4	B6		32	B ³	
		5:4			9:8
5	G ⁶		36	A^3	
		6:5		1	10:9
6	E ⁶		40	G ³	1
		4:3			9:8
8	B ⁵		45	F ³	1
		9:8			16:5
9	A ⁵		48	E ³	
		10:9			25/27
10	G ⁵		50	E ^{b3}	
		6:5		1	27:25
12	E ⁵		54	D ³	1
		5:4		1	10:9
15	C ⁵		60	C ³	
		16:15			16:15
16	B ⁴		64	B ²	1
		9:8		1	9:8
18	A ⁴		72	A ²	
		10:9			10:9
20	G ⁴		80	G ²	
		6:5			81:80
24	E4		81	g ²	
		25:24			
25	Eb4				
		27:25		1	

Fig. 2. Pitch values and ratios from regular numbers in mathematical tables.



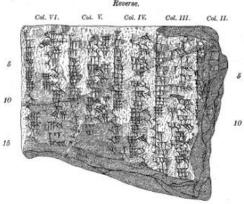


Fig. 3.1. Hilprecht's hand copies of CBM 11097, rev.

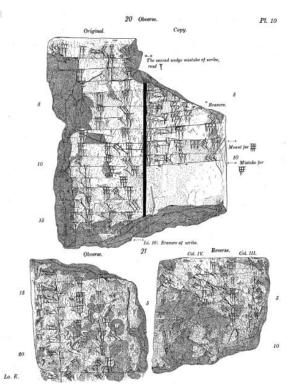


Fig. 3.2. Hilprecht's hand copies of $\it CBM$ 11340 + 11402, obv. and rev.

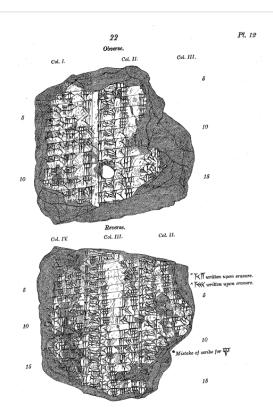
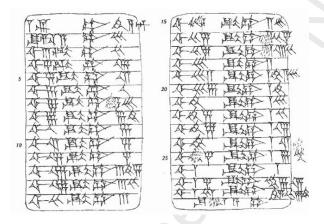


Fig. 3.3. Hilprecht's hand copy of CBM 11902, obv.



Two thirds of 1 is 0;40 Its half is 0:30	The reciprocal of 24 is 0;02 30
The reciprocal of 2 is 0;30	The reciprocal of 25 is 0;02 24
The reciprocal of 3 is 0;20	The reciprocal of 27 is 0;02 13 20
The reciprocal of 4 is 0;15	The reciprocal of 30 is 0;02
The reciprocal of 5 is 0;12	The reciprocal of 32 is 0;01 52 30
The reciprocal of 6 is 0;10	The reciprocal of 36 is 0;01 40
The reciprocal of 8 is 0;07 30	The reciprocal of 40 is 0;01 30
The reciprocal of 9 is 0;06 40	The reciprocal of 45 is 0;01 20
The reciprocal of 10 is 0;06	The reciprocal of 48 is 0;01 15
The reciprocal of 12 is 0;05	The reciprocal of 50 is 0;01 12
The reciprocal of 15 is 0;04	The reciprocal of 54 is 0;01 06 40
The reciprocal of 16 is 0;03 45	The reciprocal of 1 00 is 0;01
The reciprocal of 18 is 0;03 20	The reciprocal of 1 04 is 0;00 56 15
The reciprocal of 20 is 0;03	The reciprocal of 1 21 is 0;00 44 26 40
	Its half

Fig. 4. *MLC* 1760, Sippar? Table of reciprocals. After Clay.



Fig. 5. After Scheil, RA 12, 1915.

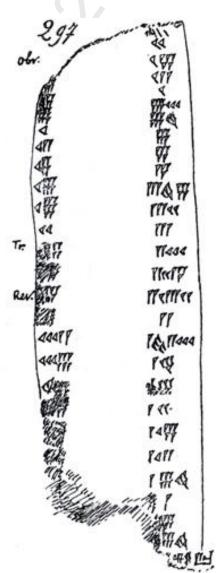


Fig. 6. After Van der Meer. 40

II - Lexical text: nabnītu 41 XXXII (UET VII,
 126 = U.3011), Obv., Cols. I and II.

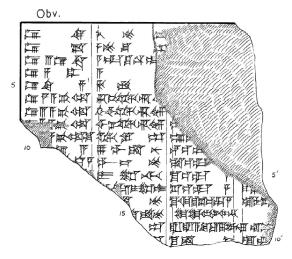


Fig. 7. Gurney's copy of UET VII 126, cols. i and ii relevant. 42

Line	Sumerian	Akkadian	Translation
1	sa.di	qud-mu-u[m	front string
2	sa.u.4	šá-mu-šu-um	next string
3	sa.3.sa.sig	šá-al-šu ga-ast-nu	third, thin string
4	sa.4.tur	a-ba-nu-/ú fourth,	small/Ea-created-string
5	sa.di.*5	ĥa-am-šu	fifth string
6	sa.4.a.ga.gul	ri-bi úĥ-ri-im	fourth behind string
7	sa.3.a.ga.gul	šal-ši úĥ-ri-im	third behind string
8	sa.2.a.ga.gul	ši-ni úĥ-ri-im	second behind string
9	[sa.1].a.ga.gul	ûĥ-ru-um	behind string
10	[9].sa.a 9	pi-it-nu	nine strings

Fig. 8. UET VII 126. Sumerian, Akkadian and translation, obv., lines 1-10. 43

It has now been safely established that this text was written during the first quarter of the first millennium. Previously, it had been dated to the Old-Babylonian Period, about 1800 B.C. because of the presence of mimation in the orthography.⁴⁴ Here, the date at which the tablet was inscribed is not as important as the period at which the original text was devised. On account of the Sumerian column, it could be construed that it had been Sumerian, dating from the third millennium B.C. During the first millennium, however, there are instances where scribes would translate their texts into Sumerian, as an exercise. But in the present text, there are inconsistencies between the Neo-Babylonian and Sumerian which might indicate that the Sumerian was indeed authentically Sumerian of the Sumerian period and not a simple first millennium translation of Neo-Babylonian into Sumerian. On this basis I would date the original text to the midthird and perhaps late-fourth millennium.

An interesting feature of the text is that it lists nine strings, unequivocally – the last line confirming it, additionally – of an undefined stringed instrument, most likely to be a large boviform lyre since there were no small occurrences of such at that period. Most large

models were boviform⁴⁵ (Fig. 9). A second interesting feature of the text is that the strings are numbered palindromically, that is 1-2-3-4-5-4-3-2-1 with locative indications, as we can read from the translation, *i.e.*, "first string of the back", etc., and others with adjectives or adjectival locutions such as "third thin-string", and "fourth string created by the god Ea".

Thus we have strings placed at the front and strings placed at the back of the instrument. But which is front and which is back is not said. It would appear logical that the front of the instrument would be at the head of the animal. However, we have no textual evidence for it and the hypothesis must remain conjectural.

The third and fourth strings of the front would have diverged from the general symmetry of the nomenclature and we shall see later with text *UET* VII, 74 that the relation of string 3, "the thin one", with its reciprocal, string 7 "of the back" was in fact a form of tritonic dissonance that was corrected by the "fourth string of the front", the string that was "corrected/made by the god Ea" who happens to be the god of music. Should we omit the 3rd and 7th string, we would have an "anhemitonic" arrangement which would have preceded, or lived alongside "diatonism" and was force-fitted into it.





Fig. 9. (Above left) The author and Jerry Baker, museum technician, carefully moving the silver lyre of Ur, from Private Grave 1237, Number U.12354 = B.M.121199, about 2600 B.C., for inspection, at the British Museum; (below) the author's replication of the silver lyre.

The origin of this fan-like disposition would, I contend, comes from prehistoric times. A solitary singer 10,000 or more years ago makes music. He may or not be accompanying himself with any instrument but what is certain is that he would start his song, probably a very simple improvisation, from a pitch where his voice was comfortable and from which he would ascend or descend as pleased him and in agreement with his mood. This would be the starting and the central note of his song that would have kept its place, much later, on the boviform lyre, as the central string.

A feature of the large lyres of the fourth and third millennia is that their smallest string is in the middle. (Fig. 10) That peculiarity would agree with both my hypothesis and the nomenclature in this text. Later, around 2600 B.C., the string plan shifted towards the player, or the back of the instrument. Thus the string plan from its original symmetric arrangement, became asymmetric, (Fig. 11) more suited to some form of diatonism.⁴⁶

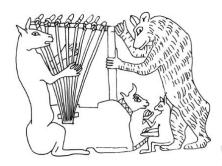


Fig. 10. Symmetric boviform lyre from Ur, ca. 2600 B.C. 47



Fig. 11. Asymmetric boviform lyre from Tello, ca. 2300 B.C. 48

There is a rare monumental lyre from Karnak in Ancient Egypt, dated around 1300 B.C., (Fig. 12) where two blind-folded musicians play, symmetrically, of the same enneachordal monumental lyre and where the central string is the shortest.

Note the presence of a small portable lyre to the right, also played by a blind-folded musician. It appears that in Ancient Egypt, both monumental and small lyre cohabitated around 1300 B.C. unlike in Mesopotamia

where at that time, large models had all but vanished. I am of the opinion that this monumental instrument was fitted with two sets of five conjunct strings sharing a central one, and that they would have been tuned anhemitonically, hypothetically g-a-c-d-e-d-c-a-g, since this arrangement would have allowed for both musicians improvising without great risk of dissonance.⁴⁹



Fig. 12. Symmetric monumental lyre from Karnak, Ancient Egypt. $ca. 1300 \, \mathrm{B.C.}^{50}$

Thus the large Sumerian boviform lyres might have initially been tuned anhemitonically, then were adapted to hemitonic diatonism which was responsible for shifting the string plan towards the player. The nomenclature would have kept the etymological traces of the historical development of the instrument. Thus the Sumerian Period might have witnessed anhemitonism and the Babylonian Period, diatonism. However, this remains conjectural and is mainly based on organological observation⁵¹ of string plans from the iconography.

This text of only ten lines is extremely rich in content. It is not a text of theory but most probably constitutes scribal observation of a musical instrument. The scribe would have asked the musician to describe his or her instrument and this is what *nabnītu* XXXII (*UET* VII, 126) is all about.

If this, in any way, shows evidence of modal expression is difficult to establish. However, some of the terms are precise and others are less so. For instance, strings 1, 2, of the front, five of the middle; 4, 3, 2 and 1 of the back would logically indicate a series of contiguous pitches. Strings 3 and 4 of the front diverge from this rule. Would this express that the variation that was brought to these two strings was only describable by imprecise terms of "thinness" and of "godly intervention?", or might these terms locate dissonance and its correction, in a tonal context, as I have already hypothesised, is difficult to assess.

If we relate the mathematical texts discussed above, to the present tablet, then the symmetry expressed within would suggest that the palindromic nine pitch set, an enneachord, would place itself in the range 36 to 81 with string length quantification of: 36; 40; 45; 48; 54; 60; 64; 72 and 81. This assumption will seem far-fetched to the enlightened musicologist. However the fourth text discussed in the present paper, *UET* VII, 74 = (U.7/80), will confirm that my assumption is, logically, correct.

There is another important clue hidden in our text. I am of the opinion that not only is it a nomenclature of strings but that additionally it shows how the enneachord was tuned. This is based on the symmetrical pattern in the nomenclature and explains the variations given to strings three and four of the front, "thin" and "Ea-created", respectively. This hypothesis is reinforced by pitch quantification given in aforementioned mathematical texts and gives justification for the presence of the number 81.

I will now explain the tuning procedure. From the central note, the axis of symmetry, labelled "5", fifths are projected toward the base and toward the treble:

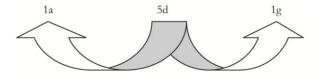


Fig. 13. Projection of fifths from string 5 defining the boundaries of the enneachord.

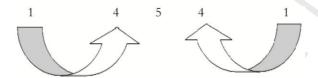


Fig. 14. Projection of fourths from the boundaries of the enneachord.

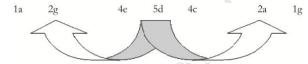


Fig. 15. Projection of fourths from central string 5.

These results in an anhemitonic tuning: a-g-e-d-c-a-g. In this construction, the tritone will place itself on 3 "of the front" and 3 "of the back" and come from the tuning of just thirds, minor and major, from the boundaries of the enneachord.

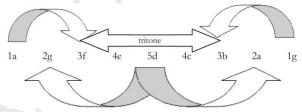


Fig. 16. The tritone results from the tuning of just major thirds (5:4) from the boundaries of the enneachord.

The placement of a tritone results in an enneachordal diatonic tuning in Just intonation as follows: 10:9; 9:8; 16:15; 9:8; 10:9; 27:25; 10:9; 9:8, which in cents is: 182; 204; 112; 204; 182; 133; 182; 204 amounting to 1403 cents, an enneachord. However, this construction poses a problem in relation to pitch quantification with regular numbers as a Just major third projected from strings 1 and 3 9 (g-b) of the back will result in the invalid figure of 64.8 since it is not an integer: it should be 64, for "b" in relation to "g". Thus the presence of 81/80 finds here its justification as should we multiply the lower Just fifths 3:2 by 80/81 we have a smaller fifth of 40:27, a grave fifth, which will correct 64.8 to 64. This discrepancy shows once more the inability in Antiquity to find an ideal system of quantification without "doctoring" figures.

Thus, the enneachord is an imperfect system when it becomes diatonic because the tritone sits on two conjunct fifths. Its construction in Just intonation requires the syntonic comma to "tame" its imperfection. Later, in Classical Greece, the imperfections of the tetrachord and of the octave were also tamed by the same mathematical devices. In the Ancient Near East, the fundamental fifth 3:2 is made up of descending 16:15; 9:8; 10:9 and 9:8 = 112; 204; 182; 204 cents, and is quantified in sexagesimal regular integers as 30; 32; 36; 40 and 45.

Thus it can be assumed that the theory of music in the Ancient Near East took the Just fifth as fundamental interval and that two such Just fifths as with *nabnītu* XXXII and with *UET*, VII, 74, and three conjunct fifths, such as with *CBS* 10996, since three conjunct just fifths amount to a triskaidecachord, expanded the system according to requirements.

Here we can witness, probably, for the first time in the history of music, the moment when theory diverges from praxis: Modality would be music that could not be notated.

CBS 10996⁵²



Fig. 17. $\it{CBS}\,10996$. The framed part only is about musical nomenclature. 53

This tablet was published by Professor Kilmer⁵⁴ in 1960. It was found at the site of Nippur and was originally thought to be from the Kassite Period, about 1500 B.C. All now agree that it is Neo-Babylonian, early first millennium B.C., but there again it is possible that this was a copy of a far older text on the basis that the terminology which it gives is known from, *UET* VII, 74, (U.7/80) dated *ca.* 1800 B.C.

It was possible to reconstruct its contents by extrapolation since there was a recurrence of two numerical patterns. For instance, the second line of the top left of the tablet (which is numbered 7, thereafter) has \text{WY} : which are the signs for 6 and 3.

The following line has w #: which is 3 and 5. Since the following line has 7-4, followed by 4-6, it was possible to find out that the beginning of line 6, $\sqrt[n]{}$ wmust be 2-4. This revealed the following pattern: 4-1/1-3; 5-2/2-4; 6-3/3-5; 7-4/4-6. The pattern then changes to 1-5/7-5; 2-6/8-6, and so forth. However this is nothing more than the invertion of the first series. In music theory the inversion of the fifth 1-5 is the fourth 4-1 where 1 is either a tonic or the octave⁵⁵. Line 7 continues with the sign SA⁵⁶ as we have seen in the previous tablet. It is followed by, kitmu. At line 11 the pattern changes and starts with the logogram SA followed by the names of the strings, that is string first and string fifth. and not string 1 and string 5 followed by the enumeration of the numbers and then by the term to which they equate as we had it from line 6 onward, i.e. 1-5 SA niš tuhri. From this it was clear that the terms following the numbers were the names given to the intervals.

It will be observed that this text was devised for a seven-stringed instrument on the basis that seven is the largest number in the tablet. Transliteration and translation of the left column, lines 11 to 24.

This schematic representation (Fig. 19) will reveal that most of the intervals listed have been inversed in order to fit within the heptachordal span. However, it is evident that this broken pattern needs to be reconstructed in order to bring back these intervals to their original pattern.

Transliteration and translation of the left column, lines 11 to 24.

Lines	Akkadian numbers and names	Translation
11	1-5 niš tuhri	rise of the equivalent
12	7-5 šēru	song
13	2-6 išartu	normal/erect/straight
14	1-6 šalšatu	third
15	3-7 embūbu	reed-pipe?
16	2-7 rebūtu	fourth
17	4-1 nīd qabli	fall of the middle
18	1-3 isqu	lot/portion
19	5-2 qablītu	middle
20	2-4 titur qablītu	bridge of the middle
21	6-3 kitmu	closing
22	3-5 titur išartu	bridge of the normal/erect/straight
23	7-4 pītu	opening
24	4-6 serdû	lament

Fig. 18. Akkadian list of numbers and names from CBS 10996, lines 11 to 25. Neo-Babylonian.

Most of my philologist colleagues have failed to understand this reconstruction and consequently have assumed and published that the Babylonians used intervals of fifths, fourth, thirds and sixths having failed to see that fourths and sixths were placed, as they are in this text, as the consequence of being adapted to a heptachordal instrument.

String	I	II	III	IV	V	VI	VII
1.11	1		niš tuhri		→ 5		
1.12					5 ◀	šēru	 7
1.13		2 ——		išartu		→ 6	
1.14	1				šalšatu	→ 6	
1.15			3 —		embū	bu	→ 7
1.16		2 ——			rebūtu		→ 7
1.17	1 🗲		nīd qabli	4			
1.18	1 —	isqu	→ 3		L,		
1.19		2 🗲		qablīt	u <u> </u>		
1.20		2 — tit	ur qablītu	→ 4		_	
1.21			3 ◀		kitmu	 6	
1.22			3 — ti	tur išartu	5		
1.23				4 ◀		pīt	ru - 7
1.24				4	serdû	→ 6	
					0.0000000000000000000000000000000000000	900 9080	

Fig. 19. Schematic representation of CBS 10996, lines 11-25 with arrows indicating the polarity.

Strings	I	II _	III	L IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
1.11	1 —		niš tuhri		▶ 5		L						
1.12				Γ	5 ◆	šēru	-7						
1.13		2—		išartu		▶ 6		L,					
1.14			1		Ĭ	6◀	šalšatu	- 8					
1.15			3—		embūbu		▶ 7						
1.16							7 ←	rebūtu	- 9				
1.17				4-	nī	d qabli		→ 8	Ľ	L			
1.18								8 ◀	isqu	-10			
1.19					5 —		qablītu		▶ 9				
1.20									9◀	titur qablītu	11		
1.21						6 —		kitmu		▶10		_	
1.22										10◀	titur išartu	-12	
1.23							7—		pītu		→ 11		L
1.24											11◀	serdû	-13

Fig. 20. Schematic reconstruction of CBS 10996. Lines 11-24.

My reconstruction showing a span of thirteen pitches agrees with the rare examples in the Sumerian iconography, exclusively, where some monumental lyres are fitted with as many strings. Elamite harps of the midfirst millennium B.C. have large spans, and as many as thirty strings can be counted. However, almost two millennia separate Sumerian lyres from Elamite harps. Hardly, these instruments can be compared on either organological, or theoretical grounds. It is most likely that when stringed instruments were produced with smaller spans that the original intervals were inversed to fit in with the smaller string plans. As we shall see with the next text, UET VII, 74 (U.7/80), dating from the Old-Babylonian Period, the intervals are inversed to fit an enneachord similar that that described in text UET VII, 126 (U.7/80).

It is impossible to say at that point of our analysis which was the direction of the pitch set. However, we can safely establish that during the Old-Babylonian Period, and probably a millennium before, the span was of thirteen pitches, and on which a series of seven fifths and seven thirds rested. Because the intervals had different names we can therefore derive that 1) they were filled and 2) that each was different from the others. That they were different is plausible because the sexagesimal quantification resulting from Just intonation has two types of tone: 9:8 and 10:9 and three types of semitones: 16:15; 24:25 and 27:24 allowing for seven genera of fifths and fourths. This is very important because it might point to the origins of the Arabian ajnās which are the building blocks of the maqāmāt. This might constitute evidence of modality in the music of the Ancient Near East.

However, the position of the thirds is not clear as it would have seemed more logical to place them, minor and major, within the fifth. Since this text dates from the first millennium, it is a possibility that they were listed as complementary to fifths to complete a heptatonic sequence. As we shall see later, text *CBS* 1766 provides

evidence of heptatonic construction and dates from 1200 to 800 B.C. $\,$

This tablet dates from the Old-Babylonian Period, about 1800 B.C. It was unearthed by Sir Leonard Woolley at Ur in the winter of 1928-29 and was published about forty years later in 1968 by the late Professor Gurney ⁵⁷. At that time no scholar had yet hypothesised that the scale may be descending.

UETVII,74 = (U.7/80)



Fig. 21. Author's photograph of the cast of *UET* VII, 74 (U.7/80)⁵⁸ with kind permission of the Trustees of the British Museum.

Consequently, Gurney's paper was presented with the assumption that the system was ascending. Then in 1982, Vitale⁵⁹ suggested that it might be descending. He was not taken seriously until the Leiden Assyriologist Th.J.H. Krispijn corrected Gurney's reading of line 12 from Sumerian *NU SU* to Old-Babylonian *nu-su-ḫu-um* from the verb *nasāḫum*, "to tighten". This new term, *nasāḥum*, Sumerian *gid-i*, or *nussuḥum*, Sumerian *zi-zi*, is the technical verb for "to tighten" strings. Its antonym is *ne'ûm*, Sumerian *tu-lu*. Enlightened by Krispijn's paper, Gurney⁶⁰ published another in 1994 in which he proposed the transliteration in Figure 22.

This fragmentary tablet is composed of two types of quatrains in the form of *protasis/apodosis*⁶¹. Type A up to line 11, included, and type B from line 13.

Type A says:

- If (protasis) the instrument is tuned in the scale of "1".
- The interval placed between strings "x" and "y" is "unclear" (tritonic?).
- **Tune up** string "x" (or "y"), (or "x" and "y").
- Then (apodosis) your instrument will be in the scale of "2".

Then after line 12 instructions are to "tune down". Quatrains of type B consist in reversing the action of quatrains type A:

- If (protasis) the instrument is tuned in the scale of "2".
- The interval placed between strings "x" and "y" is "unclear" (tritonic?).
- Tune down string "x" (or "y"), (or "x" and "y").
- Then (apodosis) your instrument will be in the scale of "1".

The formula in this text, *UET* VII, 74 (U.7/80) is symmetrical with its axis at line 12. It echoes the symmetry in *UET* VII, 126 (U.3011), around string five.

The tablet is much damaged. However, the pattern in its remaining quatrains allows for reconstructive extrapolation (Fig. 24).

This reconstruction has clear instructions: It explains how to generate scales *based on the tension and on the relaxation of strings*, exclusively (and not with ratios of string length). This means that during the Old-Babylonian Period, theory and praxis were based on Just intervals since "justness" would have been achieved when beats stopped.

In each quatrain, I) a scale is named. II) The location of "a bad sounding" interval (tritone) is located. III) Instructions are given to "tune-up" or to "tune-down" one (or two strings). IV) The outcome is a new scale. The quatrain which follows has the same instructions (for the scale that the previous quatrain generated). The quatrains succeed each other until the last which is the seventh. It generates a scale with the same name as the first one but

which is (approximately) one semitone higher than the first one.

- 1 e-e]m-b[u-bu-um la za-ku
- 2 ša-al-š[a-am qa-at-na-am tu-na-sà-aḫ-ma]
- 3 e-em bu-bu-u[m iz-za-ku]
- 4 šum-ma gišZ[À.MÍ e-m-bu-bu-um-ma]
- 5 ki-it.mu-um [la za-ku]
- 6 re-bi úh-ri-im [tu-na-sà-ah-ma]
- 7 ki-it-mu-um i[z-za-ku]
- 8 šum-ma gišZÀ.MÍ k[i-it-mu-um-ma]
- 9 i-šar-tum la za-[ka-at]
- 10 ša-mu-ša-am ù-úh-ri-a-a[m tu-na-sà-ah-ma]
- 11 i-šar-tum iz-za-[ku]
- 12 *nu-su-h[u-um]*
- 13 šum-ma gišZÀ.MÍ i-šar-t[um-ma]
- 14 qa-ab-li-ta-am ta-al-pu-[ut]
- 15 ša-mu-ša-am ù-úh-ri-a-am te-[ni-e-ma]
- 16 gišZÀ.MÍ ki-it-mu-[um]
- 17 [šum]-ma gišZÀ.MÍ ki-it-m[u-um-ma]
- 18 [i-ša]r-ta-am la za-ku-ta-am t[a-al-pu-ut]
- 19 [re-bi] úh-ri-im te-ni-e![-ma]
- 20 [gišZÀ.MÍ e-em-bu-bu-um]

Fig. 22. Gurney's 1994 transliteration of *UET* VII, 74 (U.7/80), right column.

In the order in which they appear in my reconstruction, the scales are listed on Figure 23. Some of my colleagues have advanced that these scales were not enneatonic but heptatonic on the basis that it was the instrument that was enneachordal and not the system 62. My argument against their assumption has been that in this case, what would have been the position of the heptachord within the enneachord? For instance, since enneatonic *išartum* is c-b-a-g-f-e-d-c-b, which of c-b-a-g-f-e-d, b-a-g-f-e-d-c, or a-g-f-e-d-c-b would be the heptachord? Additionally there would not have been the need for seven enneachords to host seven heptachords since each enneachord can host three heptachords.

1.	išartum	(c-b-a-g-f-e-d-c-b)
2.	qablītum	(f-e-d-c-b-a-g-f-e)
3.	niš tuhrim	(b-a-g-f-e-d-c-b-a)
4.	nīd qablim	(e-d-c-b-a-g-f-e-d)
5.	pītum	(a-g-f-e-d-c-b-a-g)
6.	embūbum	(d-c-b-a-g-f-e-d-c)
7.	kitmum	(g-f-e-d-c-b-a-g-f)

Fig. 23. *UET* VII, 74. Old-Babylonian scale system in approximate dynamic notation.

Thus, the various scales produced amount to a system of seven thetical enneatonic scales, each having its own name the nomenclature of which incomprehensible at present. On instruments with larger spans, such as third millennium boviform lyres, it is quite possible that the

scales of the system had been spread dynamically. Fifteen strings would have been needed for a system of seven enneachords. However, both the iconography and the archaeology have only but few examples of a maximum of eleven strings. Therefore the hypothesis of seven enneachords in the dynamic disposition is improbable. Would this mean that initially there were less enneachords in the system?

It is my contention that the theoreticians during the Old-Babylonian Period were facing a dilemma. The older enneatonic system, however beautifully symmetric it had been, was not suited to more "modern" concepts. While the fundamental enneachord would have been sufficient to express various "moods", our present text reveals a much more practical seven scale system. This would have progressively led to what I would call "proto"-heptatonism which finally became truly heptatonic in the first millennium B.C. as we shall see later with *CBS* 1766.

The history of music theory is not a clear cut one. Its evolution is in a way similar to the change from old weights and measures to the metric system. In France where the metric system was introduced from

1815 onward, French markets, to this day, still use premetric terms adapted to the metric system: a "livre" weighs 500 grammes. Therefore it is highly probable that the same would have applied to music terminology in the Ancient Near East with ancient terms used for new values.

There is another interesting issue arising with regard the generative tuning of the enneatonic system. It is of my opinion that it was tuned as I have explained it with text *UET* VII, 126 (projecting fifths from the central string and fourths from the boundaries of the enneachord and from the central string and then placing Just thirds) and that method would have produced the scale of *pītum*. However, it is also possible that once fifths were projected from the axis of symmetry, thirds were placed and that tones (and later, semitones) were placed approximately, or in function of modal requirements.

pītum means "opening", a term appropriate since it would "open", that is starting the scale system which, according to the mathematical texts would be: 36; 40; 45; 48; 54; 60; 64; 72; 81.

First part	Second part
1. If the harp is (tuned in the scale of) <i>išartum</i> (the interval of strings 5 and 2 is) <i>qablītum</i> sounds bad tune up string 5 Then the harp will be (tuned in the scale of) <i>qablītum</i>	1. If the harp is (tuned in the scale of) <i>išartum</i> (the interval of strings 5 and 2 is) <i>qablītum</i> sounds bad tune down strings 2 and 9 Then the harp will be (tuned in the scale of) <i>kitmum</i>
2. If the harp is (tuned in the scale of) <i>qablītum</i> (the interval of strings 1 and 5 is) <i>niš tuḥrim</i> sounds bad tune up strings 1 and 8 Then the harp will be (tuned in the scale of) <i>niš tuḥrim</i>	2. If the harp is (tuned in the scale of) <i>kitmum</i> (the interval of strings 2 and 6 is) <i>išartum</i> sounds bad tune down string 6 Then the harp will be (tuned in the scale of) <i>embūbum</i>
3. If the harp is (tuned in the scale of) <i>niš tuḫrim</i> (the interval of string 4 and 1 is) <i>nīd qablim</i> sounds bad tune up string 4 Then the harp will be (tuned in the scale of) <i>nīd qablim</i>	3. If the harp is (tuned in the scale of) <i>embubum</i> (the interval of strings 6 and 3) is <i>kitmum</i> sounds bad tune down string 3 Then the harp will be (tuned in the scale of) <i>pītum</i>
4. If the harp is (tuned in the scale of) <i>nīd qablim</i> (the interval of strings 7 and 4 is) <i>pītum</i> sounds bad tune up string 7 Then the harp will be (tuned in the scale of) <i>pītum</i>	4. If the harp is (tuned in the scale of) <i>pītum</i> (the interval of strings 3 and 7 is) <i>embūbum</i> sounds bad tune down string 7 Then the harp will be (tuned in the scale of) <i>nīd qablim</i>
5. If the harp is (tuned in the scale of) <i>pītum</i> (the interval of strings 3 and 7 is) <i>embūbum</i> sounds bad tune up string 3 Then the harp will be (tuned in the scale of) <i>embūbum</i>	5. If the harp is (tuned in the scale of) <i>nīd qablim</i> (the interval of strings 7 and 4 is) <i>pītum</i> sounds bad tune down string 4 Then the harp will be (tuned in the scale of) <i>niš tuḫrim</i>
6. If the harp is (tuned in the scale of) <i>embūbum</i> (the interval of strings 6 and 3 is) <i>kitmum</i> sounds bad tune up string 6 Then the harp will be (tuned in the scale of) <i>kitmum</i>	6. If the harp is (tuned in the scale of) <i>niš tuḫrim</i> (the interval of strings 4 and 1) is <i>nīd qablim</i> sounds bad tune down strings 1 and 8 Then the harp will be (tuned in the scale of) <i>qablītum</i>
7. If the harp is (tuned in the scale of) <i>kitmum</i> (the interval of strings 2 and 6 is) <i>išartum</i> sounds bad tune up strings 2 and 9 Then the harp will be (tuned in the scale of) <i>išartum</i>	7. If the harp is (tuned in the scale of) <i>qablītum</i> (the interval of strings 1 and 5 is) <i>niš tuḥrim</i> sounds bad tune down string 5 Then the harp will be (tuned in the scale of) <i>išartum</i>

Fig. 24. Reconstruction by extrapolation of UET VII, 74 (U.7/80).



Fig. 25. *UET* VII, 74. Old Babylonian scale system in approximate thetical notation with "tritonic" location (underlined).

However, and if I am right in assuming Just intonation tuning, then a Just third tuned from 81 would be 64.8 and not 64 as shown on Figure 27, Col. VII.

This discrepancy is interesting. If G were tuned to 80 instead of 81 then the Just major third projected from it would be 64. Consequently, the fifth V-IX = D-G would be reduced to 3:2 x 80:81 = 40:27, 680 cents, a grave fifth, and no longer the Just fifth at 702 cents. This is probably why the Just fifth was considered as the only stable interval in the whole system, along with Just thirds (minor and major) of which it is made. Thus as early as in Babylonian times, about 4,000 years ago, the syntonic comma (which I would rename "the comma of Nippur", or "Nippurian comma") corrected discrepancies as it will centuries later in Classical Greece where it will be known as the "comma of Didymus".

At that point of our research, I will put forward that this discrepancy would have partially illustrated the distinction between the "tonal" and the "modal". In Antiquity, the 'modal' would have been the tuning that was used in praxis, but that was incompatible with arithmetical rigour. Soon, mathematicians devised the "Nippurian" comma to bring order in the system, for their own mathematical satisfaction.

Musicians would have never bothered with this dilemma. They would have used, unconsciously, certain combinations of fifths which would have best suited their mood, similarly to the way in which the *ajnās* of the *maqām* tradition make up Middle and Near-Eastern scales.

Thus, theoreticians would have reduced the figure for IX of 81 to 80 in order to avoid the problematic VII at 64.8. The tables in figures 28 to 57 analyse each of the scales amounting to the Old-Babylonian system with the corrected figures and taking the scale of *išartum* as c-b-a-g-f-e-d-c-b for convenience.

The pitch set, or scale of $p\bar{\imath}tum$ would have been the scale from which all others were generated. The reason for this assumption is that firstly it agrees with the order in text UET VII, 126, secondly, this order agrees with the regular numbers in the mathematical texts discussed above. Thirdly, the term $p\bar{\imath}tum$ is particularly well suited in that it suggests that this scale was the "opening" one. Therefore, I have reconstructed the scales of UET VII, 74 from it (figures 28 to 57).

The text says that if the sammu-instrument⁶⁴ is tuned in the scale of $p\bar{\imath}tum$, then the interval between strings III and VII is "la~zaku" which we can safely interpret as being a form of "tritonic" dissonance that we shall investigate later.

Here I am cautious with the term "tritonic" which might not be appropriate because Old-Babylonian "la $zak\hat{u}$ "⁶⁵ is too vague a term to ascertain which tritonic interval it would be, *i.e.*, of what values it would be made.

pītum would be a descending enneatonic scale of 36; 40; 455; 48; 54; 60; 64; 72 and 82, which in cents would be 1382; 1200; 996; 884; 680; 498; 386; 182 and 0. Expressed in ratios, it would be: 20:9; 2:1; 16:9; 5:3; 40:27; 4:3; 5:4; 10:9; 1:1.

The intervals resulting from this construction are:

- 1. Semitones: 112 cents = 16:15 which is the Just semitone.
- 2. Tones: 182 cents = 10:9 which is the minor tone in Just intonation; and 204 cents = 9:8 which is the ninth harmonic and the major tone.
- 3. Ditones: 386 cents = 5:4, the fifth harmonic and the major third; 316 cents = 6:5 which is the just minor third; 294 = 32:27 which is the 'Pythagorean' minor third and also known in Arabian lute fretting.
- 4. Fourths: 498 cents = 4:3, the Just fourth; 520 cents = 27:20 which is the acute fourth.
- 5. Fifths: 702 cents = 3:2, the Just fifth; 610 cents = 64:45, the diminished fifth which can be taken as tritonic; (See below) 680 cents = 40:27, the grave fifth.
- 6. Tritones: 610 cents = 64:45, the diminished fifth.
- 7. Sixths: 884 cents = 5:3, the Just major sixth; 814 cents = 8:5, the Just minor sixth.
- 8. Sevenths: 996 = 16:9, the minor seventh; 1018 = 9:5, the acute minor seventh.
- 9. Octaves: 1200 cents = 2:1, the Just octave.
- 10. Ninths: 1382 = 2:1 + 10:9.

INSTRUCTIONS I

The text tells us that in the scale of $p\bar{\imath}tum$ the interval placed on strings III and VII ($emb\bar{\imath}bum$) is "tritonic" (see Fig. 28). We are instructed to tune up string III. It is not said by what amount should this string III be "tuned up". However, it is logical that it should be raised by an amount correcting the dissonance to consonance. The consonance should be Just since we are working in Just intonation where Just intervals have no interferential beats between them. Therefore string III should be "tuned up" by an amount which would make the interval between strings III and VII equal to 702 cents = 3:2.

Therefore, string III should be raised by 92 cents (since 996 + 92 = 1088, and that 1088 - 386 = 702), and the interval between strings III and VII is now a Just fifth at 702 cents = 3:2 (Fig. 29). The outcome of the instruction is that the scale of $p\bar{\imath}tum$, has now become the scale of $emb\bar{\imath}ubum$ (Fig. 30).

embūbum would be a descending enneatonic scale, which in cents would be: 1382; 1200; 1088; 884; 680; 498; 386; 182 and 0. Expressed in ratios: 20:9; 2:1; 15:8; 5:3; 40:27; 4:3; 5:4; 10:9; 1:1.

INSTRUCTIONS II

The text tells us that in the scale of $emb\bar{u}bum$ the interval placed on strings VI and III (kitmum) is "tritonic" (Fig. 31). We are instructed to "tune up" string VI. Now, the interval between VI and III = 1088 - 498 = 590 = 45:32, the tritone. Logic dictates, on the grounds of the regular numbers of the sexagesimal model, that 590 should be corrected to 498.590 - 498 = 92.92 cents = 135:128, the larger limma. VI = 498 + 92 = 590 (Fig. 32).

The outcome of the instruction is that the scale of $emb\bar{u}bum$, has now become the scale of kitmum (Fig. 33). kitmum would be a descending enneatonic scale, which in cents would be: 1382; 1200; 1088; 884; 680; 590; 386; 182 and 0. Expressed in ratios: 20:9; 2:1; 15:8; 5:3; 40:27; 45:32; 5:4; 10:9; 1:1. This scale introduces one new interval, which is a semitone: 90 = 256:243, the "Pythagorean" limma.

INSTRUCTIONS III

The text tells us that in the scale of kitmum the interval placed on strings II and VI ($i\check{s}artum$) is "tritonic" (Fig. 34). We are instructed to "tune up" string II and IX. Now, the interval between II and IX = 1200 - 590 = 610 = 64.45, the diminished fifth. Logic dictates, on the basis of the construction, that 610 should be corrected to 702. 702 - 610 = 92. 92 cents = 135.128, the larger limma. In order to correct the dissonance, 92 should be added both to 1200 and to 0 = 1292 and 92 (Fig. 35). This generates the scale of limina (Fig. 36).

This scale of *išartum* starts the second chapter of the instructions. We are now instructed to "tune down" instead of "tuning up". The system is now reversed, and

in the scale of *išartum*, strings V and II ($q\bar{a}blitum$) produce a "tritonic" interval (Fig. 37). This scale introduces a new interval, 612 = 729:512, the "Pythagorean" tritone which would have been known some 1,200 years before its ascribed inventor thought about it.

We are then instructed to "tune down" II and IX (Fig. 38). However, (1292 - 92 = 1200) and (1200 - 680 = 520) 520 = 27:20, an acute fourth, not Just by a difference of 22 cents. This generates the scale of *kitmu* (Fig. 39). In the scale of *kitmu*, the interval between strings II and VI (*išartum*) is "tritonic" (Fig. 40).

610=64:45, is a diminished fifth. We are instructed to "tune down" string VI (Fig. 41). This generates the scale of *embūbum* (Fig. 42), in which we are told that the interval between strings VI and III (*kitmum*) is "tritonic" (Fig. 43), with 590=45:32, the tritone. We are instructed to "tune down" string III (Fig. 44). This generates the scale of $p\bar{t}um$ (Fig. 45).

In the scale of *pītum* we are told that the interval between strings III and VII (*embūbum*) is "tritonic" (Fig. 46). We are instructed to "tune down" string VII (Fig. 47). This generates the scale of *nīd qablim* (Fig. 48).

We are told that in the scale of $n\bar{\imath}d$ *qablim*, the interval placed between strings VII and IV ($p\bar{\imath}tum$) is "tritonic" (Fig. 49). We are instructed to "tune down" string IV (Fig. 50). This generates the scale of $ni\check{s}$ *tulprim* (Fig. 51).

We are informed that in the scale of niš tubrim, the interval between strings IV and I ($n\bar{i}d$ qablim) is "tritonic" (Fig. 52). 600 = 140.99. This is the equal tritone.

We are instructed to "tune down" strings I and VIII (Fig. 53). This generates the scale of *qablītum* (Fig. 54). In the scale of *qablītum*, we are told that the interval between strings I and VI (*niš tuḥrim*) is "tritonic" (Fig. 55).

We are instructed to "tune down" string V (Fig. 56). This generates the scale of *išartum* (Fig. 57). The final scale of *išartum* (see below) is exactly 92 cents = 135:128 (larger *limma*) higher than the initial one (compare with Fig. 36).

Therefore the Old-Babylonian system is composed of the scales on Figure 59.

These figures, as I have mentioned before, relate to the Old-Babylonian system where the second fifth (54 - 81) is reduced to fit the quantification of 54 - 80.

Nevertheless, the inversion of the fifths due to the reduction of the original span to the enneachord might allow for extrapolating their original composition. (It goes without saying that had fifths in our texts been meant to be dyads 66 , there would not have been any logical reason to give them different names. Therefore, this should, once and for all, settle the argument spearheaded by Professor Kilmer that the intervals were empty. It is of my opinion that these fifths were filled as they are in the *ajnās* of the *maqāmāt* 67 which stem, I believe, from the Old-Babylonian material 68 . Text *CBS* 10996 includes ditones.

These would of course follow the same rule as the fifths and would have been filled since they have, also, different names.

Seven descending diatonic fifths each starting from successive diatonic of a diatonic descending generative pitch sequence would necessarily include a tritone at some point depending with which fifth the sequence started. So it seems logical that the sequence ended with the tritonic fifth. We have a lack of evidence regarding the

dating of the intervallic nomenclature and therefore it is impossible to say if it came before pitch quantification had been instituted – if it ever were.

Thus although it may appear logical that different names of fifths defined fifths differing in their morphology, it is impossible to define their structure securely. However, should we rely on the logical structure of our texts, then the seven fifths would be as shown in Figure 59.

120								000									
number	I		П		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3	0	2		1
ratio	A	10:9	G	9:8	F	16:15	Е	9:8	D	10:9	С	27:25	В	10:9	A	9:8	G
q.value*	36		40		45		48		54		60		64.8		72		81

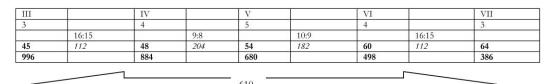
^{*} quantification value

Fig. 26. pītum tuning.

cents	1382		1200		996		884		680		498		386		182		0
q/i value*	36	182	40	204	45	112	48	204	54	182	60	112	64	204	72	182	80
ratio		10:9		9:8		16:15		9:8		10:9		16:15		9:8		10:9	
order	1		2		3		4	- 7	5		4		3		2		1
pītum	I		II		III		IV		V		VI		VII		VIII		IX

^{*} q / i value means quantification and intervallic value (in cents)

Fig. 27. Reconstruction of the scale system in UET VII, 74, in cents.



996 cents - 386 cents = 610 cents

Fig. 28. Instructions from pītum.

1088		884		680		498		386
Cent value	204		204		182		112	
	9:8		9:8		10:9		16:15	
3		4		5		4		3
III		IV		V		VI		VII

Fig. 29. Instructions.

embūbum	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio		10:9		16:15		9:8		9:8		10:9		16:15		9:8		10:9	
i.value		182		112		204		204		182		112		204		182	
cents	1382		1200		1088		884		680		498		386		182		0

Fig. 30. Scale of embūbum.

III		IV		V		VI
3		4		5		4
	9:8		9:8		10:9	
i.value	204		204		182	
1088		884		680		498

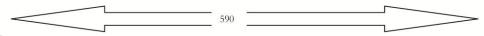


Fig. 31. Instructions: the interval placed on strings VI and III (kitmum) is "tritonic".

III		IV		V		VI
3		4		5		4
	9:8		9:8		256:243	
i.value	204		204		90	
1088		884		680		590

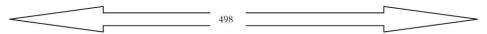


Fig. 32. Instructions: logic dictates that 590 should be corrected to 498, on the basis of sexagesimal metrology of regular numbers.

kitmum	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio		10:9		16:15		9:8		9:8		256:243		9:8		9:8		10:9	
i.value		182		112		204	8	204		90		204		204		182	
cents	1382		1200		1088		884		680		590		386		182		0

Fig. 33. Instructions: outcome = the scale of kitmum.

II		III		IV		V		VI
2		3		4	Î	5	T T	4
	16:15		9:8		9:8		256:243	
i.value	112		204		204		90	
1200		1088		884		680		590

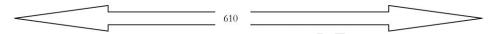


Fig. 34. Instructions: the interval placed on strings II and VI ($\it i\~sartum$) is "tritonic".

II		III		IV		V		VI
2		3	1	4		5		4
	9:8		9:8		9:8		256:243	
i.value	204		204		204		90	
1292		1088		884		680		590

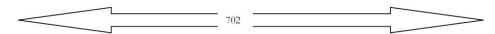


Fig. 35. Instructions: "tune up" string II and IX.

išartum	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio		256/243		9:8				9:8		256:243		9:8		9:8			
i.value		90		204		204		204		90		204		204		90	
cents	1382		1292		1088		884		680		590		386		182		92

Fig. 36. Instructions: the scale of išartum (1).

II		III		IV		V
2		3		4		5
	9:8		9:8		9:8	
	204		204		204	
1292		1088		884		680

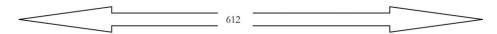


Fig. 37. Instructions: "tune down" producing a "tritonic" interval.

II		III		IV		V
2		3	λ.	4		5
	16:15	T _C	9:8		9:8	
	112		204		204	
1200		1088		884		680

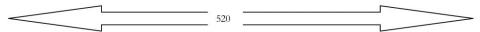


Fig. 38. Instructions: "tune down" II and IX.

kitmu	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio		10:9		16:15		9:8		9:8		256:243		9:8		9:8		10:9	
q.value		182		112		204		204		90		204		204		182	
cents	1382		1200		1088		884		680		590		386		182		0

Fig. 39. Instructions: the scale of kitmu.

II		III		IV		V		VI
2		3		4		5		4
	16:15		9:8		9:8		256:243	
40	112		204		204	54	90	
1200		1088		884		680		590

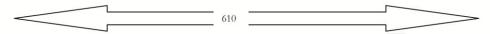


Fig. 40. Instructions: the interval between strings II and VI (išartum) is "tritonic".

II		III		IV		V		VI
2		3		4		5		4
	16:15		9:8		9:8		10:9	
40	112		204		204		182	
1200		1088		884		680		498

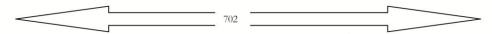
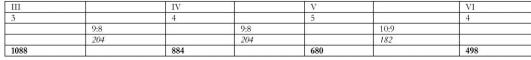


Fig. 41. Instructions: "tune down" string VI.

							_				-					_	
embūbu	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio		10:9		16:15		9:8		9:8		10:9		16:15		9:8		10:9	
q.value		182		112		204		204		182		112		204		182	
cents	1382		1200		1088		884		680		498		386		182		0

Fig. 42. Instructions: the scale of $emb\bar{u}bum$.



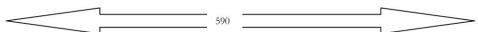


Fig. 43. Instructions: the interval between strings VI and III (kitmum) is "tritonic".

III		IV		V		VI
3		4	<u> </u>	5		4
			9:8		10:9	
	112		204		182	
996		884		680		498

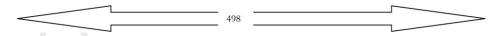


Fig. 44. Instructions: "tune down" string III.

pītum	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio		10:9		9:8		16:15		9:8		10:9		16:15		9:8		10:9	
q.value		182		204		112		204		182		112		204		182	
cents	1382		1200		996		884		680		498		386		182		0

Fig. 45. Instructions: the scale of pītum.

996		884		680		498		386
	112		204		182		112	
	16:15		9:8		10:9		16:15	
3		4		5		4		3
III		IV		l V		VI		VII

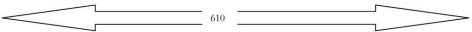


Fig. 46. Instructions: the interval between strings III and VII ($emb\bar{u}bum$) is "tritonic".

III		IV		V		VI		VII
3		4		5		4		3
	16:15		9:8		10:9		9:8	à
	112		204		182		204	
996		884		680		498		294

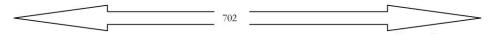


Fig. 47. Instructions: "tune down" string VII.

nid qablim	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio	c#	10:9	Ь	9:8	a	16:15	g#	9:8	f#	10:9	e	9:8	d#	16:15	c#	10:9	Ь
q.value		182		204		112		204		182		204		112		182	
cents	1382		1200		996		884		680		498		294		182		0

Fig. 48. Instructions: the scale of nīd qablim.

IV		V		VI		VII	
4		5		4		3	
	9:8		10:9		9:8		
	204		182		204		
884		680		498		294	

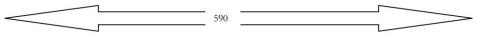


Fig. 49. Instructions: the interval between strings VII and IV ($p\bar{t}um$) is "tritonic".

IV		V		VI		VII	
4		5		4		3	
	16:15		10:9		9:8		
	112		182		204		
792		680		498		294	

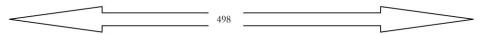


Fig. 50. Instructions: "tune down" string IV.

niš t.	I		II		III		IV		V		VI		VII		VIII	c 9.	IX
order	1		2		3		4		5		4		3		2		1
ratio		10:9		9:8		9:8		16:15		10:9		9:8		9:8		10:9	
q.value		182		204		204		112		182		204		204		182	
cents	1382		1200		996		792		680		498		294		182		0

Fig. 51. Instructions: the scale of niš tuhrim.

I		II		III		IV
1		2		3		4
	10:9		9:8		9:8	T T
	182	40	204	45	204	
1382		1200		996		792

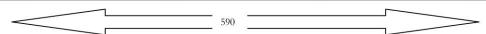


Fig. 52. Instructions: the interval between strings IV and I ($n\bar{l}d$ qablim) is "tritonic".

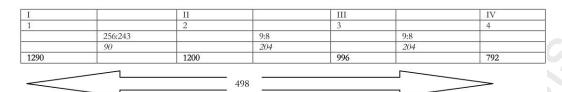


Fig. 53. Instructions: "tune down" strings I and VIII.

qablitum	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2		3		4		5		4		3		2		1
ratio		256:243		9:8		9:8		16:15		10:9		9:8		9:8		256:243	
q.value		90		204		204		112		182		204		204		90	
cents	1290		1200		996		792		680		498		294		90		0

Fig. 54. Instructions: the scale of qablitum.

I		II		III		IV		V
1		2		3		4		5
	256:243		9:8		9:8		16:15	
	90		204		204		112	
1290		1200		996		792		680

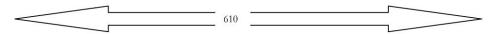


Fig. 55. Instructions: the interval between strings I and VI (niš tuḥrim) is "tritonic".

I		II		III		IV		V
1		2		3		4		5
	256:243		9:8		9:8		9:8	
	90		204		204		204	
1290		1200		996		792		588

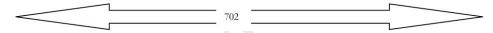


Fig. 56. Instructions: "tune down" string V.

isartum	I		II		III		IV		V		VI		VII		VIII		IX
order	1		2	3	3		4		5		4		3		2		1
ratio		256:243		9:8		9:8		9:8		256:243		9:8		9:8		256:243	
q.value		90		204		204		204		90		204		204		90	
cents	1290		1200		996		792		588		498		294		90		0

Fig. 57. Instructions: the scale of išartum (2- Compare with Figure 36).

			Т	une up					
	I	II	III	IV	V	VI	VII	VIII	IX
išartum	1290	1200	996	792	588	498	294	90	0
qablītum	1290	1200	996	792	680	498	294	90	0
niš tuhrim	1382	1200	996	792	680	498	294	182	0
nīd qablim	1382	1200	996	884	680	498	294	182	0
pītum	1382	1200	996	884	680	498	386	182	0
embūbum	1382	1200	1088	884	680	498	386	182	0
kitmum	1382	1200	1088	884	680	590	386	182	0
išartum	1382	1292	1088	884	680	590	386	182	92
			Tu	ne down					
išartum	1382	1292	1088	884	680	590	386	182	92
kitmum	1382	1200	1088	884	680	590	386	182	0
embūbum	1382	1200	1088	884	680	498	386	182	0
pītum	1382	1200	996	884	680	498	386	182	0
nīd qablim	1382	1200	996	884	680	498	294	182	0
niš tuhrim	1382	1200	996	792	680	498	294	182	0
qablītum	1290	1200	996	792	680	498	294	90	0
išartum	1290	1200	996	792	588	498	294	90	0

Fig. 58. The Old-Babylonian system.

NT 1 .	A	. 1
Nomenclature Nomenclature	<u>Approximative</u>	pitch
	11	1

1.	niš tuhrim	$E^4 - D^4 - C^4 - B^3 - A^3$
2.	išartum	$D^4 - C^4 - B^3 - A^3 - G^3$
3.	embūbum	$C^4 - B^3 - A^3 - G^3 - F^3$
4.	nīd qablim	$B^3 - A^3 - G^3 - F^3 - E^3$
5.	qablītum	$A^3 - G^3 - F^3 - E^3 - D^3$
6.	kitmum	$G^3 - F^3 - E^3 - D^3 - C^3$
7.	pītum	$F^3 - E^3 - D^3 - C^3 - (B^{b3})$

Fig. 59. Nomenclature of fifths.

It will be immediately obvious that the seven fifths listed above are distinct from one another. This would be the justification for their different names in support of my assumption that intervals were filled and not dyads.

It is my contention that the differences in the aforementioned morphology of fifths would have been much more expressive than that allowed by the sexagesimal regular numbers. This is reflected in the nomenclature which would have distinguished them as nowadays maqāmāt nomenclature immediately suggests the mood of the piece to be played, even by non musicians. There is also the probability that the morphology of fifths would additionally have been dependant on the composition of the piece, on the emphasis of the phrase, on the interpretation of the musician and on many other factors in a manner comparable to inflections in poetic recitation, and perhaps in Judaic cantilation, Christian chant, traces of which may be surviving in the Byzantine material, and later in Coranic declamation which all might find their sources in the Babylonian material.

CBS 1766



Fig. 60. *CBS* 1766. Courtesy of the University Museum of the University of Pennsylvania, Philadelphia.

This text cannot be dated with accuracy as it was also acquired by the University Museum of the University of

Quantification

24(9:8)	27(10:9)	30(16:15)	32(9:8)	36
27(10:9)	30(16:15)	32(9:8)	36(10:9)	40
30(16:15)	32(9:8)	36(10:9)	40(9:8)	45
32(9:8)	36(10:9)	40(9:8)	45(16:15)	48
36(10:9)	40(9:8)	45(16:15)	48(9:8)	54
40(9:8)	45(16:15)	48(9:8)	54(10:9)	60
45(16:15)	48(9:8)	54(10:9)	60(16:15)	(64?)

Pennsylvania out of archeological context. However, on the basis of its contents I would place it around the turn of the second and first millennia B.C.

The contents are of enormous importance as they constitute the first ever recorded evidence of a truly and unequivocally heptatonic construction based on the alternation of fifths and fourths, a well as the description, possibly, of a device which would have located the seven scales of a heptatonic diatonic system.

The text is composed of a graphic representation of a heptagram⁶⁹ inscribed in two concentric circles, and of tables with a majority of unfilled, or unreadeable columns and rows. However, the columns that can be read hold essential integers. The header of the table is also unreadable although some attempts have been proposed.⁷⁰

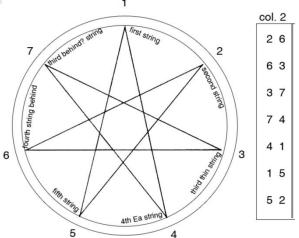


Fig. 61. Reconstruction of the essential elements of the text.

Any musicologist presented with a heptagram would conclude, should they be assured that the context is musical, that the figure is the diagrammatic construction of a heptatonic diatonic musical scale. They would expect to find numbers, notes, pitches or degrees on each of the points of the star, starting from the top, and then conclude that the intersecting lines linking the numbers indicate the alternation of intervals of fifths and fourths which are the basis for the formation of the diatonic heptatonic paradigm.

Should they wish to illustrate further the principle, they would draw a table with a series of numbers which would flow in the following sequence: 1-5-2-6-3-7-4-1, as complementary explanation of how the heptagonal construction works. Should they substitute notes for numbers, as they are displayed on the circumference, clockwise, then the notes could be any ascending or descending series starting on any note of the heptatonic scale: c-d-e-f-g-a, or b. It is therefore unsurprising that the names and numbers which appear on the heptagon in CBS 1766 are precisely what our music theoretician would have written, without hesitation, on a similar pattern. Indeed, the number at the top of the heptagon is 1 and its nomenclature is qu-ud-mu, meaning "first string", unsurprisingly. The orthography diverges from UET VII 1267. There we have Sumerian sa.di with Akkadian equation qud-mu-u/m. The second term, clockwise, is headed with number 2 followed by sa-mu*šum*, close enough to *sa-mu-šu-um* in the same *UET*; the term which follows is not readable but it must have been šal-al-šu qa-at-nu since this is what follows in our text of reference; then we have a-banu rightly followed by haan-šu and re-bi? uh-ri. The sequence ends with number 7, šal-šu [XX]. The last signs resist reading but we would expect something expressing that it was the xth behindstring, i.e., the xth last string as we have it in *UET* VII, 126. Now, that we have both the names of the strings as well as numbers on the heptagram is of high significance as this constitutes the first instance in the history of music of a dichotomy between the string itself and the sound it produces.

Thus the heptagram has both nomenclature and number. This is evidential of a system where we have a scale of the first degree, of the second degree, of the third degree, etc., starting on the first string, on the second, and so forth. The number of strings is now restricted to seven, depicting both heptachordal and heptatonical systems as basis for this new theory.

This text marks a radical change in music theory as it exposes a preference for the tonal rather than for the modal. Indeed, there is no mention of names of fifths or thirds which would be "proto-*ajnās*", no more enneatonic scales related to the morphology of fifths, and perhaps of thirds that were revealed in older texts such as UET VII, 126 and 74.

The 29 tablets of which one only will be discussed in the present paper were unearthed during the pre and post war Missions at Ras Shamra conducted by the French scholar Claude Schaeffer. They are written in the Hurrian language with syllabic Babylonian cuneiforms and date from about 1400 B.C. The scribes who wrote these texts were Akkadians or Semites with Akkadian fluency accounting for the Hurrianisation of the original Akkadian terminology.

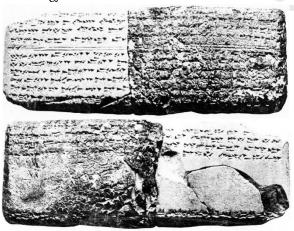


Fig. 62. Photograph of H.6. Courtesy of the Museum of Damascus.



Fig. 64. Hand copy by the author of the cuneiform text above.

The tablets, in all probability, would all have had the same rectangular shape to fit the length of the hand. The writing runs parallel to the longest side and is divided in three. The first part varies with each tablet but generally the text spreads onto the obverse. The text usually consists of one paragraph which ends by a double line, with a double Winkelhaken⁷¹ at the beginning and at the end, on the obverse. The second part spreads below the double line and consists of Hurrianised Akkadian musical terms which are followed, in most cases, by a number and sometimes preceded, or followed, by a qualificative. The first part gives the verse and the second the music and rhythm. A colophon, which constitutes the third part, runs along the bottom edge of the tablets and states that it is... a song in the "mode" of "x" followed by a qualificative and deities to whom the hymn is devoted. Then follows the name of a scribe, a certain Ammurabi; another, Ipšali and the name of one of four Hurrian composers: Tapšihun, Puhiyanna, Urhiya, Ammiya.

Regrettably, the tablet which I am presenting here is the only tablet, reconstructed from three fragments, which came reasonably intact to us, H.6 = (R.13.30 + 15.49 + 17.387). For the purpose of this paper, I shall not discuss the upper register of the text because it

consists in the lyrics of the song which is of no particular importance in the present context.

The music part of the text which is inscribed below the double line has the text shown in Figure 65.

A first observation shows that each line includes six terms with the exception of line 6. However it is possible that a sixth expression existed at the end of this line because the surface is very damaged. It would be unreasonable to assume that this last term differs from the others simply because it is unreadable. Furthermore, that the other lines include 6 terms would tend to favour the presence of a sixth one there. Thus we may assume that each line included six terms as shown on Figure 66.

Each terms is followed by a number with the possible exception of the fourth of the first line (5-IV); the last one in the first (5-VI); the last in the second (6-VI) and last in the tenth line (10-VI). However, the surface is damaged and there is no reason to assume that these terms were not followed by a number.

- 5. qablite 3 irbute 1 qablite 3 rxx xxx7 titimišarte 10 uštamari
- 6. titimišarte 2 zirte 1 šahri 2 zirte 2 irbute 2?
- 7. umbube 1 šaššate 2 irbute [1] šaššate 2? titarqabli 1 titimišarte 4
- 8. zirte 1 šahri 2 šaššate 4 irbute 1 nadgabli 1 šahri 2?
- 9. šaššate 4? šahri 1 šaššate 2 šahri 1 šaššate 2 irbute 2
- 10. kitme 1 qablite 3 kitme 1 qablite 4 kitme 1

Fig. 65. Musical intervals and rhythm in H.6.

	I	II	III	IV	V	VI
5	qablite 3	irbute 1	qablite 3	XXXXX	titim išarte 10	uštam ari
6	titimišarte 2	zirte 1	šaḥri1	zirte 2	irbute 2	XXXXX
7	umbube	šaššate 2	irbute 1	šaššate 2	titarqabli 1	titimišarte 4
8	zirte 1	šahri 2	šaššate 4	irbute 1	natqabli 1	šahri 2
9	šaššate 4	šahri 1	šaššate 2	šahri 1	šaššate 2	irbute 2
10	kitme 2	qablite 3	kitme 1	qablite 4	kitme 1	qablite x

Fig. 66. Hypothetical reconstruction of the musical notation.



Fig. 65. The author's reconstruction of the music in H.6.

The colophon says that the piece is written in the "mode" of *nidqibli*, the descending enneatonic scale D: E-D-C-B-A-G-F-E-D. Since all the surviving colophons of the collection indicate the usage of the scale of *nidqibli*, to the exclusion of any other, it is possible that the series was composed in the same model. Therefore we can assume that *qablite* in the first line equated to A-(g-f-e)-D which is a descending fifth and that all other intervals in the text followed the same principle. The interpretation of the hapax legomenon *uštamari* has not yet been discussed but it could be assumed that it was a term for another interval, perhaps different from the fifth or the third and that it could also have been followed by a number. Since the tablets contain both text and music it makes little doubt that the musical notation was accurate enough to

match the exactness of the syllabic arrangement in the text. A less accurate rendition would have served no purpose.

Now, the tablet is the notation of a song. It says so in the colophon. There are no indications as to any form of instrumental accompaniment. As far as we know, the voice cannot produce dyads, simultaneously. This means that the intervals were filled in order to support the lyrics. Had the music been reduced to dyads, sung consecutively, the purpose for the nomenclature of fifths and fourths and their invertion would had had no purpose whatsoever. The contents of these filled intervals would have had specific melodic, and possibly, modal values.

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Notes

¹ A *Modus vivendi* is an agreement between parties of diverging opinions. When two sides reach a *modus vivendi* regarding cultural incompatibilities, an accommodation of their respective differences is established for the sake of contingency.

² About the myth of Pope Gregory the First's ascribed reform: John the Deacon's complaint about Frankish barbarism comes from his biography of Gregory. St. Gregory compiled a book of antiphons using the contemporary term for a kind of liturgical singing. He founded a *schola* using the contemporary term for a choir which to this day performs the chant in the Church of Rome according to his instructions; he also erected two dwellings for it at St. Peter's and at the Lateran palace, where are venerated the couch from which he gave lessons in chant, the whip with which he threatened the boys, and the authentic *antiphonarium*, the latter being the great book containing the music for the whole liturgical calendar. See [Taruskin, 2010, Chapter One: The curtain goes up].

³ Mŏdus, i, m. root med-, measure, weigh; Gr. μέδομαι, μέδοντες, μήστωρ, μέδιμνος, cf.: modius, modestus, moderor, I. a measure with which, or according to which, any thing is measured, its size, length, circumference, quantity (freq. and class.). The measure of tones, measure, rhythm, melody, harmony, time; in poetry, measure, metre, mode: "vocum," Cic. Div. 2, 3, 9: "musici," Quint. 1, 10, 14: "lyrici," Ov. H. 15, 6: "fidibus Latinis Thebanos aptare modos," Hor. Ep. 1, 3, 12: Bacchico exsultas (i. e. exsultans) modo, Enn. ap. Charis. p. 214 P. (Trag. v. 152 Vahl.): "flebilibus modis concinere," Cic. Tusc. 1, 44, 106: "saltare ad tibicinis modos", to the music or sound of the flute, Liv. 7, 2: "nectere canoris Eloquium vocale modis," Juv. 7, 19 – Fig.: "verae numerosque modosque ediscere vitae," moral harmonies, Hor. Ep. 2, 2, 144 (abbreviations of the names of authors and other abbreviations, signs, etc. can be found in [Smith, 1855, p. ix–xi]).

⁴ [Picard, 2001].

⁵ [Field, 1998].

⁶ The earliest codex preserving ancient Greek music theory is Heidelbergensis Palatinus gr. 281. It was probably written in Seleucia on the west bank of the Tigris River, Mesopotamia (present day Iraq) by the scribe, Nikolaos Kalligraphos, and completed on January 14, 1040. The manuscript is preserved at Heidelberg University Library. The scribe's colophon states that "this book was assembled from many works among the private papers of Romanus, judge at Seleucia and my master. All you who read it, pray for him". The codex was conceived as a complete book; there are no blank leaves or sides. It preserves Psellus' complete Syntagma together with the preliminary Logices, and this is followed by his Opiniones de anima, a short excert from Leontinius on the hypostatases, Chapter 38 from Photius Quaestiones ad Amphilochium, and ten short theological treatises by Theodore Abucara, an author represented in Arethas' collection of books. It is surely no coindicence that this codex preserves these particular works, which point back to libraries of the ninth century, as well as the work of Psellus. After Theodor Abucara, the codex includes the koine hormasia and an accompanying canon; three sections from Theon of Smyrna's treatise, here titled "Division of the Musical Canon"; a short explanation of the musical ratios and genera, part of which corresponds to section 103 of the so-called Bellermann's Anonymous, and a series of excepts from Bacchius treatise ... - see [Mathiesen, 1992]. The earliest surviving sources date from the 11th century and most are later. Accordingly, while it is still possible to trace the filiation of surviving sources through, at times, a fairly closed recension, the chronological gap between author and earliest source must be considered in dealing with the content of the material. Another concern is the large amount of musical material which

appears in works that themselves are not primarily musical in content; these works include not only general encyclopaedias, but also works of philosophy (Plato's Timaeus is perhaps the best-known case), poetry, drama, and other forms of literature. Finally, the existing catalogues of this source material have become outdated and remain lamentably incomplete. The only prior attempt to provide a complete index was compiled by Karl von Jan ([Jan, 1899]). It has been necessary to resort to individual library catalogues, listings of sources in other editions, scattered footnotes, and other diffuse channels to find sources needed. ⁷ The Suda is a 10th century Byzantine encyclopaedia of the ancient Mediterranean world. formerly attributed to an author (erroneously) known as Suidas. It is an encyclopaedic lexicon, written in Greek, with 30,000 entries, many drawing from ancient sources which have since been lost, and often derived from medieval Christian compilers. The derivation is probably from the Byzantine Greek word "souda". meaning "fortress" or "stronghold", with the alternate name, Suidas. stemming from an error made by Eustathius, who mistook the title for the proper name of the author.

⁸ By the end of the fourth century C.E., ancient Greek music theory was merely part of the residue of an ancient civilization and the distinctions among the traditions were blurred or forgotten. It remained for writers such as Martianus Capella, Boethius, and Cassiodorus – all of whom relied on relatively late sources – to preserve and transmit the little that remained to the Latin readers of the Middle Ages. Thus, later Greek writers such as Nicomachus, Ptolemy, Gaudentius, and Aristides Quintilianus represent both the final stages of Greek music theory in antiquity and, as filtered through their Latin interpreters, the first stages of ancient Greek music theory as it came to be known in the Middle Ages – see [Mathiesen, 2006], *in extenso*.

⁹ However, the nature of some texts infers that the knowledge within may be sourced to far earlier times and it is my contention that embryonic music theory might have developed alongside numeracy before it expanded into literacy. Therefore I would argue that some form of music theory, probably the consequence of observation, would have been known in the fourth millennium B.C. This hypothesis is based on organological iconography and of its possible survival in nomenclature.

 $^{\rm 10}\,\mbox{In}$ 1960, a young philologist, Anne Drafkorn-Kilmer, was given a tablet to study. It turned up to be a text with musical theory. This is how a philologist with no musicological background became the "leading expert" in that field. Whenever musicologists attempted at publishing their researches in Assyriological periodicals, they were rejected on the basis that it was not musicological material, and whenever the papers were proposed to musicological reviews, they were equally rejected, this time on the basis that it was Assyriological material. Thus musicology "proper" has been segregated from Assyriology for many years and it was only in 2008, at the first international conference of ICONEA (International Conference of Near Eastern Archaeomusicology) held at the British Museum and jointly organised by Richard Dumbrill (ICONEA) and Irving Finkel (Middle East Department of the British Museum) that for the first time musicologists and Assyriologists met at last, for a coherent discourse in that field.

¹¹ An agglutinative language is a language which uses agglutination extensively. Most words are formed by joining morphemes together. This term was introduced by Wilhelm von Humboldt in 1836 to classify languages from a morphological point of view. It is derived from the Latin verb agglutinare, which means "to glue together". In agglutinative languages, each affix typically represents one unit of meaning (such as "diminutive", "past tense", "plural", etc.), and bound

morphemes are expressed by affixes (and not by internal changes of the root of the word, or changes in stress or tone). Additionally, and most importantly, in an agglutinative language affixes do not become fused with others, and do not change form conditioned by others (see [Bodmer, 1972, p. 53]).

¹² Semitic languages are a group of related languages the living representatives of which being spoken by more than 270 million people across much of the Middle East, North Africa and the Horn of Africa. They constitute a branch of the Afroasiatic language family. The most widely spoken Semitic languages today are Arabic with 206 million native speakers, Amharic with 27 million, Hebrew with about 7 million, Tigrinya, 6.7 million, and Aramaic with about 2.2 million. Semitic languages are attested in written form from a very early date, with texts in Akkadian appearing from around the middle of the third millennium BC, written in a script adapted from Sumerian cuneiform. However, most scripts used to write Semitic languages are abjads — a type of alphabetic script which omits some or all of the vowels, which is feasible for these languages because the consonants in the Semitic languages are the primary carriers of meaning. Among them are the Ugaritic, Phoenician, Aramaic, Hebrew, Syriac, Arabic, and South Arabian alphabets. The Ge'ez alphabet, used for writing the Semitic languages of Ethiopia and Eritrea, is technically an abugida - a modified abjad in which vowels are notated using diacritic marks added to the consonants (according to [Bennett, 1998; Bergsträsser, 1995; Garbini, 1984] and others).

 13 Any object which is not adequately recorded during an academic archaeological excavation cannot be appropriately dated, and of course, located unless the object itself is explicit of both date and location, which is extremely rare. In the $19^{\rm th}$ century there has been a number of "grave robbers" who plundered and sold their bounty to often less than honourable "antiquarians". These, in turn, frequently enhanced the objects by formidable descriptions. These objects were sold to wealthy private collectors keen on acquiring bits of the past, as was fashionable at that time. Most $19^{\rm th}$ century private collections are therefore unreliable (see note No. 26).

¹⁴ [Galpin, 1937, p. 99–104].

¹⁵ Scribal reliability is not attested, as far as we know, on any cuneiform tablet, *i.e.* there are no texts describing it. However, evidence of reliability and accuracy lies within the texts themselves. When scribes copied a tablet, they noted every detail of the original, that is where there had been an erasure, or where the tablet had been damaged or where part of it had broken away. They even replicated orthographic errors. There is a tablet in the Collections of the British Museum where the scribe had punched three quarters of the tablet with his stylus and written "broken", "broken", "broken", etc., all over. This quality remained consistent throughout the usage of cuneiforms that is about three millennia.

¹⁶ [Hilprecht, 1906]. Hermann Volrath Hilprecht was born in 1859 in Germany. In 1882, he spent two months in the British Museum studying cuneiform literature. He received his Ph.D. from Leipzig in 1883. In 1886 he left for the United States, where he became a professor of Assyrian at the University of Pennsylvania. He participated in the first campaign of excavations at Nippur, Iraq in 1889. With announcing the discovery of the Temple Library of Nippur after finishing the fourth campaign, some other team members including the former expedition director John Punnett Peters built a strong opposition against Hilprecht who claimed "the cream" of nearly every important discovery as his work. Some American orientalists joined in and the so called "Peters-Hilprecht-Controversy" was born.

¹⁷ Bousquet, Margaux, forthcoming. See [Schiel, 1925]. Schiel was born on June 10, 1858, in Koenigsmacker and died on the September 21, 1940 in Paris. He was a French

Dominican scholar and Assyriologist. He was one of the discoverers of the Codex Hammurabi in Persia. He took courses of Egyptology and Assyriology at the École des Hautes Études in Paris. He then undertook a series of archaeological missions in the Middle East, and in Egypt.

¹⁸ [Crickmore, 2013].

¹⁹ nabnītu s.; 1. offspring, progeny, product, 2. habitat, place of growth, 3. living creature, 4. appearance, stature, features; MB, Bogh., SB; cf. [Roth, 1965].

 20 Sir Charles Leonard Woollev (17 April 1880 – 20 February 1960) was a British archaeologist best known for his excavations at Ur in Mesonotamia. He is considered to have been one of the first modern archaeologists, and was knighted in 1935 for his contributions to the discipline.

²¹ [Kilmer, 1960].

²² [Kilmer, 1965, p. 261–272].

²³ [Gurney, 1973].

²⁴ [Kilmer, 1960] op. cit.

²⁵ [Gurney, 1968; 1994; Vitale, 1982].

 26 This is one of the collections the origins of which are of uncertain provenance (see note No. 13).

 $^{\rm 27}$ [Nougayrol, 1955; Schaeffer, 1955; Schaeffer, 1962a; Schaeffer, 1962b].

 28 A forerunner of Just Intonation since tonal quantifications in the regular numbers of the sexagesimal system are multiples of the sides of the right angle triangle.

²⁹ [Dumbrill, 2009].

³⁰ These text had not been published because scholars did not yet understand them and mostly consisted in U.7/80 and U.3011, which would later be known as UET VII, 74 and UET VII 126, respectively.

31 In the Babylonian sexagesimal notation, the reciprocal of a regular number has a finite representation. Specifically, if n divides 60^k , then the sexagesimal representation of 1/n is just that for $60^k/n$, shifted by some number of places. For instance, suppose we wish to divide by the regular number $54 = 2^{1}3^{3}$. 54 is a divisor of 60^{3} , and $60^{3}/54 = 4000$, so dividing by 54 in sexagesimal can be accomplished by multiplying by 4000 and shifting three places. In sexagesimal $4000 = 1 \times 3600 +$ $6\times60 + 40\times1$, or (as listed by Joyce) 1:6:40. Thus, 1/54, in sexagesimal, is $1/60 + 6/60^2 + 40/60^3$, also denoted 1:6:40 as Babylonian notational conventions did not specify the power of the starting digit. Conversely $1/4000 = 54/60^3$, so division by 1:6:40 = 4000 can be accomplished by instead multiplying by 54 and shifting three sexagesimal places. In Babylonian music theory, the just a pitch set involves regular the pitches have frequencies proportional to the numbers in the sequence 24, 27, 30, 32, 36, 40, 45, 48, 54, 60, 64, 72, 80, 81, of nearly-consecutive regular numbers. Thus, for an instrument with this tuning, all pitches are regularnumber harmonics of a single fundamental frequency. This pitch set is called a 5-limit tuning, meaning that the interval between any two pitches can be described as a product 2p3q5r of powers of the prime numbers up to 5, or equivalently as a ratio of regular numbers.

32 [Hilprecht, 1906, p. 21].

33 [Adams, 1902, v. VIII, p. 264–318].

34 [Dumbrill, 2009], op. cit.

³⁵ "Dans une collection nous avons: ¹ igi x gal-bi, etc. traité par Hilprecht (Babyl. Exped., XX, 1, p. 22-29) et à sa suite par Pinches (Hilpr. Anniv. Vol., p. 71-78). Il nous semble que l'on peut donner à ces textes un sens plus obvie que n'ont fait ces auteurs. Pourquoi

vaudrait-il 12.960.000 et non pas sa valeur documentée de 60. Rien n'oblige en lisant le quotient dans DA 2/3 = € de donner à chacun des quatre la valeur de 2.160.000, sinon la fiction que égale 12.960.000. Notre texte, en effet, qui n'est pas postérieur à Hammurabi distingue nettement entre les signes ∜ et ∜ . Le premier se trouve dans la liste des diviseurs, rev. 4, 5, 6: igi € gal-bi, igi € 🖫 galbi, igi € IIII gal-bi, etc., où il ne peut se lire respectivement que pour 40, 40 + 5, 40 + 8, - et dans la liste des quotients à la première ligne: da liste des quotients, l. 8, 12, rev. 8, 10, 11. Le signe ≪ signifiant certainement 40 dans la liste des diviseurs, rev. 4, 5, 6, il est impossible que dans ce même texte, colonne voisine, le signe \(\varphi\) signifie 40 – de la même manière. Moins encore, l'expression du quotient ∜ de la ligne 1 peut-elle rien avoir de commun avec celle des quotients: "Te qu'en donnait à $\frac{4}{3}$ sa valeur fractionnaire sinipu = 2/3 = 40/60, cf. Brunn. 10026, 10027; et sa valeur d'unités: 40 à ¥, signe que le scribe aurait aussi employé plus loin, s'il s'agissait d'unités majeures : 600 x 4 ou 36000 x 4 ou 2160000 x 4. Dès lors, si le quotient de la ligne 8, par exemple, est 6 40/60 ou 6 2/3 (dans le système décimal 6.666...) – il suit que le quotient de la première ligne est bien de 40 et que le dividende général de la table n'est point 12.960.000, mais simplement 60. Plus tard, les scribes confondront les deux signes et emploieront exclusivement \(\forall \), sans gêner pour cela le calcul des contemporains. A priori, d'ailleurs, et sans soulever d'autres objections, est-il croyable que sur des tablettes d'exercices scolaires, et propres à être consultées comme modèles, des gens pratiques, tels que l'étaient les Babyloniens, se soient livrés à ce jeu de chiffres qui fait jongler avec des millions, milliards, trillions, etc.? En quoi cela pouvait-il faire d'eux des 'accomplished arithmeticians'? C'est une opinion fausse que dans les groupes de chiffres d'un produit, les derniers doivent toujours être des unités, les précédents des unités d'ordre supérieur, jusqu'à = 12.960.000 (et rien n'empêche de pousser au delà!) Pourquoi méconnaître que les Babyloniens connussent et sussent exprimer exactement ou approximativement à tout degré des fractions d'unités? Au lieu que, dans le système décimal, de la subdivision de l'unité principale en parties successives de 10 en 10 fois plus petites - résulte ce que nous appelons fractions décimales, ainsi dans le système sexagésimal usité par les Babyloniens, les opérations analogues donnent des fractions sexagésimales – non seulement dans les tablettes astronomiques à valeurs angulaires ou horaires, mais en général dans tout calcul."

- ³⁶ [Hilprecht, 1906, p. 25].
- ³⁷ In a private phone communication with Leon Crickmore.
- 38 [Roth, 2010a].
- ³⁹ Babylonian and Assyrian gods were also known by numbers. Anu, the principal god was 60 or 1, since they counted in sexagesimal arithmetics; Enlil, was 50 and Ea, the god of music and of measure was 40 and also the god of two thirds. Sin was 30, and Shamash 20. There were other god complementing the system, for instance Inanna was represented by 15. We have here all the elements of a system based on just intonation. See [Dumbrill, 2005, p. 35; Labat, 1976, p. 243; Livingstone, 1986, p. 30–48] and [Röllig, 1995]; [Dumbrill, 2007]
- ⁴⁰ Scheil in [Mecquenem and Scheil. Mission de Susiane, 1935].

- 41 Due to the italics used for the title, the term $nabn\bar{\imath}tu$ is written here in standard font.
- 42 The tablet appears in Ur Excavations Texts. Publications of the joint expedition of the British Museum and of the University Museum of the University of Pennsylvania, Philadelphia, to Mesopotamia. Volume VII, Middle Babylonian Legal Documents and other Texts. Oliver Robert Gurney. Note the lacuna in 1.4, col.2 which was later corrected in IRAQ XLVI 82, note 1. Professor Gurney writes back to me on this matter on the $15^{\rm th}$ April 1996: "... I must have left the end of the line for a second look because it was dirty or otherwise difficult to read and then forgotten to come back to it. This happened to me several times!".
- ⁴³ Extracted from [Dumbrill, 2005, p. 27].
- ⁴⁴ In Old Babylonian, words in the singular have an ending on *-m*, typically *-um*, *-im*, *-am* respectively in the nominative, genitive and accusative case. This is called mimmation after the Semitic pronunciation *mim* of the letter m. It never carries the word accent. The mimmation is lost after the Old Babylonian period.
- ⁴⁵ [Dumbrill, 2005, p. 234–249]
- ⁴⁶ Here, the term "diatonism" is used *cum grano salis*, as it is of course impossible to say what was the tuning, or the tonal(?) system of an instrument from its iconographic representation. However, if one agrees that the strings would have had the same mass and the same tension, then it is possible to hypothesize that, for instance, the morphology of the large harps of the third millenium at Mari and elsewhere (See Dumbrill in the appendix to [Marcetteau, 2010, p. 73–75]). Therefore, the disposition of the string plan in our example here is more suited to "a form of" diatonism than it is suited to "a form of" anhemitonism. Quantifications for both diatonism and anhemitonism cannot be extrapolated and therefore remain assumptive.
- $^{\rm 47}$ Line drawing by Higano, Yumiko, in [Dumbrill, 2005, p. 34, Pl. 6 & p. 246, Pl. 25].
- $^{\rm 48}$ Line drawing by Higano, Yumiko, in [Dumbrill, 2005, p. 247, Pl. 26].
- ⁴⁹ In a recent verbal communication with Bruno de Florence, it was discussed that the musicians in this scene might not have played together, that is simultaneously, but perhaps consecutively, in a responsorial or imitation form. The lapicide would have decided to depict both musicians playing rather than one and not the other, waiting for his turn. Here the term dissonnance must be taken cautiously as it is impossible to qualify dissonance without knowing to what system the term refers to. However, in the case of some form of anhemitonism and in relation to the practice in particular ethnomusicological contexts, notably in the Cameroon, it can be construed that anhemitonism is more suited to "natural or just" consonance see [bakabeyond, 2009], etc.
- ⁵⁰ See [Manniche, 1991, p. 91, 54].
- ⁵¹ Although Amorite iconography has evidence of anhemitonism in its instrumentarium. See Dumbrill's appendix to [Marcetteau, 2010].
- 52 Catalogue of the Babylonian Section of the collection of cuneiform texts of the University Museum of the University of Pennsylvania, Philadelphia (unpublished).
- $^{\rm 53}$ Author's photograph with kind permission of the University Museum of the University of Pennsylvania.
- 54 [Kilmer, 1960].
- ⁵⁵ The word "unison" refers to two notes either of the same frequency or distant by one or more octaves from each other. It is the simultaneous execution of one polyphonic part by more than one performer or performing group (e.g. the first violin section of an orchestra), either at exactly the same pitch or at the interval of an

octave, double octave etc.; such execution is said to be "in unison" and is often indicated in scores by the Italian *all'unisono* (see [Wikipedia Contributors, 2012]).

- ⁵⁶ SA is a Sumerogramme which translates as *pitruu* in Akkadian: *pitruu* s.; 1. string of a musical instrument, 2. (a stringed musical instrument); [LÚ].NAR *ina pi-it-ni* [...] the musician on the *pitruu* [praises you] see [Roth, 2005].
- ⁵⁷ [Gurney, 1968].
- 58 This is a cast of the original tablet which has been returned to the Iraqi Museum in Baghdad in the 70s. Whether the tablet has survived the two wars (Iraq 1991 and 2003) is not yet known.
- ⁵⁹ [Vitale, 1982].
- 60 [Gurney, 1994].
- ⁶¹ In linguistics, a *protasis* is the subordinate clause (the if-clause) in a conditional sentence. For example, in "if X, then Y", the *protasis* is "if X". The other clause ("then Y") is called the *apodosis*. In logic, the *apodosis* corresponds to the consequent, the *protasis* to the antecedent.
- 62 [Hagel, 2009].
- 63 Didymus introduced a distinction in the diatonic tetrachord between a major and minor whole tone (respectively 9:8 and 10:9). The major and minor whole tone together constitute a major 3^{rd} (5:4), previously found only in the enharmonic tetrachord of Archytas; and in including a major 3^{rd} , the diatonic tetrachord of Didymus resembles the upper or lower tetrachord of the modern major scale (e.g. C–D–E–F, or G–A–B–c. This tetrachord was adopted by Ptolemy, but with the positions of the major and minor whole tones reversed, as his "tense" diatonic tetrachord. The difference between the major and minor tones (9:8 \times 9:10 = 81:80) is known as the "syntonic comma", or "comma of Didymus"; this is also the difference between the Pythagorean major 3^{rd} (81:64) and the pure major 3^{rd} (5:4).
- ⁶⁴ See [Roth, 2010b]. I render occurrences of it as sammû-instrument, because in my opinion based on organological evidence, during the old-Babylonian Period there were no lyres fitted with as many as nine strings. However, there is ample evidence that vertical harps, during the same period, were fitted with as many as nine strings. (For a comprehensive description of lyres and harps during the periods mentioned, see [Dumbrill, 2005]) Additionally, a harp in which strings are better approportionated to a diatonic scaling would be a better instrument for the application of the instructions in this text, rather than a lyre where the strings have little variation in length. Therefore the sammû-instrument should be a "harp". The reason for my decision not to translate sammû with "harp" is that in UET VII, 74, it is written with Sumerian ggZA.MÍ, "gg" being the determinative for "wood" indicating of which predominant material the instrument was built) and that this might indicate a Sumerian origin for the instrument under scrutiny which therefore would be a lyre rather than a harp.

However, it might possible be that the scribe, for some unexplained reason, had opted for the usage of Sumerograms on that occasion for reasons that are obscure, but not uncommon.

- 65 See [Roth, 1961]. It is highly probable that the Old-Babylonian did not have a proper musicological term for describing the tritonic dissonance. This is perhaps because it could be found either in the fifth or in the fourth. However, the term with the negative la, clearly means "unclear", "not pure", etc., and is sufficient to indicate a tritonic dissonance in the present context.
- 66 The term "dyad" is used by Kilmer to imply that in any interval know in the Sumero-Babylonian nomenclature, only the first and the last note is sounded. This remains her postulation.
- ⁶⁷ Trichords are sets of 3 notes, tetrachords are sets of 4 notes, and pentachords are sets of 5 notes. The Arabic word for these sets is jins, plural ajnās, which means the gender, type or nature of something. In case of pentachords, the word 'aqd, plural 'uqūd, is also used. These sets are the building blocks for Arabic magām(s). It is possible and often practical to view a magam as a collection of sets, as well as a collection of notes. Each magam is made up to two main ajnās (sets) called lower and upper jins. The lower jins is used to group or classify the magam in a family. In general the starting note of the upper jins is called the dominant note. A maqām also includes other ajnās (called secondary) which overlap the two main ajnās, and can be exploited during modulation. Different Arabic music references define sets slightly differently. As with maqām(s), many sets are too archaic or rarely used. There is also disagreement about the length of each set (3, 4, or 5 notes), and some references simplify and standardize every set as a tetrachord. In general all sets are defined as tetrachords unless there is a good reason not to. A set is a defined as a trichord when the next (4th) note is impossible to predict out of multiple choices, as in the Sīkā and Musta'ār trichords for example. Another reason to define a set as a trichord is when 3 notes are enough to convey its melody or mood. An example of this are the 'Ajam and Jahārkā trichords. Complex sets (containing other partial sets) are defined as pentachords, as in Nawā-Athar and its variation Athar-Kurd for example (see [Maqam World, 2004]).
- 68 On the basis of their structure as each interval of the fifth is different.
- 69 In general, a heptagram is any self-intersecting heptagon, a sevensided polygon. There are two regular heptagrams: 1) the 7/2heptagram and 2) the 7/3 heptagram. It is the 7/3 heptagram which is depicted in CBS1766. This is the smallest star polygon that can be drawn in two forms, 7/2 and 7/3, as irreducible fractions.
- ⁷⁰ [Dumbrill, 2008]; [Friberg, 2008]; [Horowitz, 2006].
- 71 The *Winkelhaken* (German for "angular hook", also simply called "hook" in English) is one of five basic wedge elements appearing in the composition of signs in Akkadian cuneiform. It was realized by pressing the point of the stylus into the clay.