THE TRUTH ABOUT BABYLONIAN MUSIC

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"In describing non-western music, be it oriental or primitive, one must strictly refrain from misusing incongruous concepts of western music. The terminology that has been learned in music school applies to the harmonic structure of music and is inappropriate, indeed misleading and distorting in descriptions of non-harmonic, non-western music"

[Curt Sachs – *The Wellsprings of Music*][1]

INTRODUCTION[2]

The title of this article was not meant to benefit from the current world trend of "untruths" with which we are saturated.[3] It is coincidental but nevertheless opportune. Truly, there has been a chain of "untruths" about the history of Babylonian music theory for the past fifty-seven years, and growing strong. I will address the matter, its causes, consequences and remedy.

The tablets examined in this article are the oldest texts of music theory ever found anywhere in the world. They were published from the early 1960s onward as the corpus increased when new texts were discovered[4] (mainly in the museums where they were kept). Authors approved each other's interpretations with meaningless addenda. According to their authors, the Babylonian scale could only be ascending, tense diatonic,[5] heptatonic and octavial because for them, "it could not be anything else".

Then, in 1994, a paper[6] fuelled by the new reading of a verb, turned the world upside-down and all, or most, agreed that the scales were descending.[7]

Another paper[8] claimed, extraordinarily, that the intervals listed on a tablet were to be played simultaneously, a view resting on no evidence, as there is, to my knowledge, no known comparable system, anywhere in the world, past and present. This dogma met with the horns of dilemma with Kilmer's interpretation of the Hurrian "hymn":[9] Which of the two notes to sing?

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[1] [Sachs, 1962, p. 49].

[2] This article is an edited version of the original draft temporarily published as a print on demand document.

[3] Barry, D., quoting adviser to president Trump, who used "alternative fact" to describe assertions by the White House: "In a Swirl of "untruths" and "falsehoods", calling a lie a lie. *The New York Times*, Jan 25, 2017, https://www.nytimes.com/2017/01/25/business/media/donald-trump-lie-media.html.

[4] *nabnītu* xxxii; CBS 10996; U.7/80; YBC 11381; CBS 1766; H6 (RS13.30 + 15.49 + 17.387). These references are well known to Assyriologists. They refer to various collections: CBS = Catalogue of Babylonian Section of the University Museum, Philadelphia, Pennsylvania; YBC stands for Yale Babylonian Collection; H refers to the tablets of Ras Shamra. *nabnītu* xxxii is the name of a series of tablets. The xxxii, is also known as UET VII 126, standing for Ur Excavation Texts, volume VII, Plate 126.

[5] It is clear that these authors did not know of other diatonisms. Obviously, they meant that it was "ditonic diatonism" to which they referred as it addresses the only type in Western theory. Beyhom, in his Hypothesis [Beyhom, 2017] uses "ditonic" to differentiate tense diatonism which has two Pythagorean tones in the Just fourth, from other types of diatonism.

[6] [Gurney, 1994, p. 101–106].

[7] Scales, or pitch sets can be read in an ascending manner: *c-d-e-f-g-a-b-c* or in a descending manner: *c-b-a-g-f-e-d-c*. These are simple changes of their polarity. The scale *b-a-g-f-e-d-c* is the reciprocal of f-g-a-b-c-d-e. Reciprocity is ruled by the cyclic construction of the scales. Descending *b-a-g-f-e-d-c* results from the alternation of descending fifths and ascending fourths (*b-e-a-d-g-c-f*) while, ascending *f-g-a-b-c-d-e* results from the alternation of ascending fifths and descending fourths (*f-c-g-d-a-e-b*). Therefore *b-a-g-f-e-d-c* is the reciprocal of *f-g-a-b-c-d-e*.

[8] [Duchesne-Guillemin, 1963].

[9] See Chapter VI, H6: The "proof of the pudding"?

The problem was solved, laboriously, when Kilmer "spin-doctored" the matter and decreed that the paired pitches, were the accompaniment of the hymn, and that either bass or treble pitch of the dyad could be chosen to make up the melody, a rather bizarre method. It usually is the melody which commands the accompaniment, and not the contrary.[10]

Furthermore, the colophon does not mention any instrument, a fact dismissed by Kilmer.

Many assumptions laid unfounded, with their authors dismissing, and even censoring every article challenging their views.[11] Regardless, these flaws crept into encyclopedias and other publications.[12]

For their analysis of Ancient Near-Eastern musicology, these scholars, mostly had used Western musicological tools. Most systems can be explained by, and made to fit heptatonism, for example, by squeezing, metaphorically, pitches on and between the lines of the stave, by explaining scale constructions with elusive alternations of fourths and fifths. But none of these experts were willing to accept that other methods also do exist.

Their insistence at force-fitting a musical system into the Western model, and in this case with the "unconscious" aim at acculturating Semitic[13] musicology under the Occidental yoke, is nothing but a last breath, it is hoped, of supremacist musicology. It is one of the greatest oversights in the history of music. It came from the methodology (or rather of its absence) of certain Assyriologists and of their determination at spearheading "their discovery" by means of unsuitable Western models. A bit like translating Old-Babylonian with a grammar of Mandarin.

The manner in which systems are constructed, whether consciously or not,[14] are part of the culture of a people and must be unveiled with the utmost respect and without linkage to theories of later cultures as this would lead to colonialist unification.

This article is the consequence of my determined endeavor at academically fostering the proof of the evidence against unproven presumptive inference, and more significantly to assert, scientifically, that heptatonism[15] – which is not universal – is by no means engraved onto mankind's unconscious. It is a structure, among others, which eventually hatched in the Near-East, as part and consequence of another or other systems, but not as a new, independent and exclusive concept.

This work is intended for a general readership. Therefore, Assyriologists and musicologist may find some of my explanations facile. I have avoided diacritic signs for Akkadian and Arabic transliterations whenever possible. I have used the English language notation *c-d-e-f-g-a-b* so that readers may have an approximation of the musical sets and sub-sets described on the basis that Babylonian intonation, while different, is sufficiently close to our Western practice. I have avoided as often as possible mentioning musical ratios as while meaningless to

[10] [Hagel, 2005]. Here, Hagel authoritatively writes that Babylonians only could notate accompaniment but not melody! I quote [p. 290]: "It is significant that this system was not orientated towards melody, as was Ancient Greek notation and music theory, but to instrumental practice".

[11] Madame le Docteur Marcelle Duchesne-Guillemin warned me, (in other diplomatic terms in a private correspondence) against publishing anything contradicting the current interpretations of Sumero-Babylonian music theory.

[12] See for example [Kilmer and Mirelman, 2001].

[13] I am using the term "Semitic" in its etymological meaning and not relating, exclusively to the Jewish people as it is nowadays.

[14] "Qu'il ait fallu en quelque sorte ce quelque chose qu'est l'analyse, et qui est venu nous annoncer qu'il y a du savoir qui ne se sait pas, et que c'est à proprement parler un savoir qui se supporte du signifiant comme tel […]" – Jacques Lacan, *Séminaire 20, Encore, séance du 20 mars 1973*, in [Lacan, 1975], available in audio as http://www.valas.fr/IMG/mp3/lacan-encore-20-mars-1973.mp3. This loosely translates as: "Analysis has come to announce that there is a sort of knowledge that is not known and which is based on the signifier as such […]" – [Lacan, 1999]. There is an on-going dispute between two schools of thought, the first conditioned by dogmatic *a prioricity* (see [Field, 1998]) which sustains the irrational belief in the universality of Western diatonism(s). This position finds reasonable, firstly to infer anything without any empirical evidence, infallibly, because in this case nothing can be taken as evidence against it, and that therefore (undefined) diatonism must be the consequence of "just intervals theory", or of the "theory of resonance", or for the reason that "it cannot be otherwise than it is". The second school of thought is animated by objective *a posterioricity* which opposes *in toto* the dogmatic *a prioricity*. Both schools are therefore mutually exclusive and therefore obsolete in the rhetorical discourse.

[15] There are various cultures where instruments sets are tuned in precise intervals without any construction and unconsciously memorized – [Sachs, 1962, p. 103] for a detailed tuning method.

many, they are subjectivist tools unsuited to the epistemology of Babylonian musicology.

Most obsolete musical terms are replaced with neologisms which will be explained whenever they appear or whenever necessary.

Whenever possible, I have avoided naming researchers in the body of the text. They are acknowledged in footnotes.

My usage of the following terms: dyads (2), triads (3), tetrads (4), pentads (5), hexads (6), heptads (7), octads (8), and enneads (9), etc., define "containing intervals" having pitches inside them, *i.e. C-d-e-f-G*, where *C* and *G* are the boundaries of the container and *d-e-f*, the infixed pitches.

They differ from seconds, thirds, fourths, fifths, sixths, sevenths (heptachords), eights (octaves), etc., which are empty cells used in heptatonic tuning constructions, or for general theoretical and practical purposes.

I use the terms "infix" to qualify pitches placed within intervals of triads and pentads. Intervals larger than pentads are made up of smaller intervals, for example a hexad is made of two conjunct intervals, a triad and a tetrad. Pitches placed before the principal infix or "nucleus", are called infrafixes, and those above are suprafixes.

It is the many possible locations of infixes, diverging from constructed pitches (*i.e.* such as pitches resulting from the alternation of just fifths and just fourths) which define the cultural source of a given set.

These structures erroneously became known as "modes", a term which only appeared during the Dark Ages of the Christian West and are only suitable for ecclesiastical types.

The theory of music is a science developed by, and made up for the amusement of the musicologist and is of little concern to the musician. However, Mesopotamian musicology is unique because its earliest reporters – the scribes – laid the fundaments of theory from their meticulous observation of the lyre, probably, and of its strings, and comments from the musician's mouth.

As such, it has drawn the most accurate portrait of pre- and early literate music, a feat never achieved before and since, in the long history of music.

I - *NABNĪTU*[16] XXXII: SETTING THE STRINGS

This text was excavated by Sir Leonard Wooley at Ur, Southern Iraq, in the late 1920s.[17] It dates from the middle of the first millennium BC and might be the copy of an older text, perhaps Old-Babylonian, from the early second millennium BC, and possibly earlier, I think, because of musicological and philological hints. It is a bilingual lexical text where the left column is written in Sumerian and the right, its translation, in Babylonian (Table 1). Most importantly, the text also reveals, in a second layer of meaning, an implied tuning pattern for a structure made of two conjunct pentads, amounting to an enneadic set or scale of nine pitches.

Line	Sumerian	Akkadian	Translation
1	sa.di	*qud-mu-u[m*	front string
2	sa.uš	*šá-mu-šu-um*	next string
3	sa.3.sa.sig	*šá-al-šu qa-a[t-nu*	third, thin string
4	sa.4.tur	*a-ba-nu-[ú*	fourth, small/Ea-created-string
5	sa.di. 5	*ḫa-am-[šu*	fifth string
6	sa.4.a.ga.gul	*ri-bi úḫ-ri-im*	fourth behind string
7	sa.3.a.ga.gul	*šal-ši úḫ-ri-im*	third behind string
8	sa.2.a.ga.gul	*ši-ni úḫ-ri-im*	second behind string
9	[sa.1].a.ga.gul	*úḫ-ru-um*	behind string
10	[9].sa.a	9 *pi-it-nu*	nine strings

Table 1 Transliteration and translation of Sumerian and Babylonian terms in *nabnītu* xxxii.

There are ten lines. The tenth says "nine strings". This indicates, I contend, inconspicuous indications for the harmonic interaction of nine strings. It has been advanced that the Sumerian word "sa = string", Akkadian "*pītnu*"[18] (with qualifiers such as "di", "2.a.ga.gul"[19], etc.) excluded the pitch to which a string was tuned.

[16] The word translates as either 1) offspring, progeny, product, living creature, 2) habitat, place of growth, 3) living creature, 4) appearance, stature, features. *Chicago Assyrian Dictionary* (see [Roth, 2012]), *CAD* henceforth, Vol. "n".

[17] [Gurney, 1974], Pl.74.

[18] *CAD*, Vol. "p". (See note 16 above)

[19] Sumerian sa.di; sa.2.a.ga.gul. "sa" = "string/pitch", "di" means "foremost, prime". "2.a.ga.gul" means "second of behind".

I would find it illogical that a Babylonian theoretician segregated the pitch of a string from its name in his demonstration, which otherwise would be pointless. Therefore, the word "pitch" is a substitute for "string", and reciprocally. The practice remains today, as the "*e*" string of a violin is called the "chanterelle" in French.

In the English language, the "*e*"; the "*a*"; the "*d*" and the "*g*" strings of the violin are tuned to "*e*", "*a*", "*d*", and "*g*" respectively. In Bach's "*g*"-string Air, it is the string and the piece which take the name of the pitch. May I remind the reader that the seven strings of the Greek lyre had names which became synonymous to the pitches of the scale, in descending order.[20] There is no reason why this would not have been inherited from a Babylonian precursor, but it is even more surprising that scholars did not make this parallel.

These nine strings (therefore pitches), are consistently mentioned in texts from the second to the first millennium BC. This means that for two thousand years, and perhaps more, a nine-pitch system was known. However, I do not suggest that a nine pitch or enneadic (bi-pentadic)[21] scale was the only one in practice during that period. I am of the opinion that there were concurrent structures. Sumer and Babylon, had different counting systems for different things and therefore it would not be dazing should music, too, have conformed to different ones. Additionally, there would have been various regional styles adding to the sound palette. These regionalisms persist to this day in rare countries which have not yet been polluted by equal temperament, or where regionalisms are protected.

For extrapolation, I will propose that the interval between strings 1 and 2 of the front has the same value as the interval between strings 2 and 1 of the back. The interval between strings 2 and 3 of the front has the same value as the interval between strings 3 and 2 of the back. The interval between strings 3 and 4 of the front has the same value as the interval between strings 4 and 3 of the back and finally, the interval between strings 4 and 5 of the front has the same value as the interval between strings 5 and 4 of the back. Therefore, the intervals between strings 1^{f}-3^{f} and 3^{b}-1^{b} are equal; between strings 1^{f}-4^{f} and 4^{b}-1^{b} are equal and between 1^{f}-5 and 5-1^{b} are also equal. This is probably why the strings were recorded in this palindromic manner.

The nine strings should be read as $1^{f(front)}$-2^{f}-3^{f}-4^{f}-5-$4^{b(ack)}$-3^{b}-2^{b}-1^{b} but never 1-2-3-4-5-6-7-8-9, as most scholars did, because this would imply that the scale is heptatonic, with two added pitches, which it is not. The scale is made up of two conjunct pentads[22], such as: a-g-f-e-d/d-c-b-a-g.

The pattern can be simplified as:

1 2 3 4 5 4 3 2 1

with 5 in red, as pitch of conjunction.

Strings 3 and 4 (green) of the front have terms to qualify them. These Sumerian qualifications vary in their Akkadian version. The reason for this will be explained later as it is essential to Babylonian theory. Another important philological detail is that the first string is called "sa.di" in Sumerian and so is the fifth string called "sa.di.5", with added "5". If "di" means "prime" as well as "first", then "di" emphasizes the value (in the theory) of strings one and five (1-5-1) because they are the boundaries of the system. The Babylonian translation does not reflect this.

Modern Western music uses the equal temperament system (ET)[23] where tones and semi-tones measure 200 and 100 cents respectively.[24] They

[20] This will be evident to the Hellenist since the names of the strings were also the names of the notes. This fact is given in most books about Greek music, for example in [West, 1992], p. 64.

[21] As we shall see later, an ennead or set of nine contiguous pitches (tense diatonic) is made up of two pentadic subsets.

[22] The numbering of the strings from one to nine led to the conclusion that it was heptatonic, with strings eight and nine being at the octave of strings one and two, but since the set is made up of two conjunct pentads, neither pentad can accommodate octaves.

[23] Composers do not imagine their music in Equal Temperament. It is far removed from their creativity. However, in order to make their music playable, the transposition of the imagined music is written with it. My concern is that computer programs used by modern composers, have forced their creations into an ET infrastructure, not unlike composing "at the piano" has, in its time, contributed to the melodic enslavement to the harmonic master.

[24] The cent is a logarithmic unit used for measuring musical intervals. Twelve-tone equal temperament divides the octave into 12 semitones of 100 cents each. Cents are used to quantify or to compare intervals. Alexander J. Ellis based the measure on the acoustic logarithms decimal semi-tone system developed by the French mathematician Gaspard de Prony in the 1830s. See [Ellis, →

are ascending, heptatonic, octavial, and (tense) diatonic, for example: *c-d-e-f-g-a-b-c*, a scale of C major. They are made up of tones and semi-tones arranged in a strict sequential order. For the purpose of tonal appreciation, the symmetry in *nabnītu* xxxii, can be played with our modern scale extended to nine pitches, for example: *g-a-b-c-d-e-f-g-a*, or its descending form: *a-g-f-e-d-c-b-a-g*. But it must be borne in mind that this translation is only approximate because it is constructed from a different method. To the untrained ear, the scales played one after the other would sound very similar, but would reveal differences when played together.

In Fig. 1 below, columns in grey indicate tone[25] intervals; yellow, semi-tone intervals. The red column is the axis of symmetry of the system.

1		2		3		4		5		4		3		2		1
a		g		f		e		d		c		b		a		g
g		a		b		c		d		e		f		g		a
	tone		tone		1/2		tone		tone		1/2		tone		tone	

Fig. 1 Position of tones and semi-tones in the enneadic/bi-pentadic system of *nabnītu* xxxii.

Although they would have had the mathematical ability,[26] it is very improbable that Babylonians used the equal temperament. The size of their intervals would have differed slightly, but significantly, from our Western systems. It is my opinion that they used Just Intonation because it is the most natural manner to produce and appreciate intervals, at least in theoretical musicology, but it would certainly not have been an inflexible rule.[27]

A Just Intonation[28] fifth measures 702 cents, (expressed by the ratio of 3:2)[29] (=701.955001 cents); a Just Intonation fourth measures 498 cents, (expressed by the ratio 4:3) (=498.044999 cents), etc. In the Equal Temperament fifths measure 700 and fourths 500 cents, respectively.

From this basis, it is possible to build up an estimation of how the generative[30] Babylonian set might have sounded, but first, I shall describe the implied tuning process.

Firstly, the central string is tuned to an appropriate pitch. This will depend on the quality of the string. From my own experiments with sheep gut-strings, such a string sounds its best when stretched at about 80% of its breaking point. Therefore, it is possible to make an estimation of pitch in relation to the type of string used

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1876] – notably p. 9-11 – and, for more information on Prony, [Anon. "Gaspard de Prony", 2016].

[25] The word "tone" is a term used to designate an unqualified interval, *i.e.*, an interval which can be Just, Pythagorean, ET, etc. Sachs writes [1962, p. 60–61] that "tonic" has six different meanings. 1) As an adjective used as a noun, it is the main gravitational pole of a harmonized or harmonizable melody. The original Greek noun, *tonos* (and hence, via Latin and Old French, our "tone" is related to "tension" and means, 2) acoustically speaking, any regular sound as opposed to irregular noises; 3) the pitch, vibration number, or frequency of such a sound, say *C* or *C* sharp; 4) its colour or timbre, warm or cool; 5) a melody pattern (like "psalm-tone", and 6) the distance or interval of a major second.

[26] [Fowler and Robson, 1998] explain, in the abstract, [p. 366]: "We consider several aspects of the role and evaluation of the four-sexagesimal-place approximation to √2 on the well-known Old-Babylonian tablet YBC 7289. By referring to what is known about OB school texts, we show that this text is most probably a school exercise by a trainee scribe who got the approximation from a coefficient list. These coefficient lists are briefly described, with their use in geometrical problems. We consider other texts involving square roots and derive an algorithm for evaluating them, which complies with all known OB examples, from a simple geometrical construction of the type that seems to underlie many other OB procedures". Therefore, they would have been able to calculate an equal temperament scale. However, there must be a

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distinction between the ability as an unconscious knowledge, (unknown known) and the need to apply such a concept when the application is possible because of the ability. "Si, avec un si, on peut mettre Paris dans une bouteille, on doit pouvoir aussi, avec un si bémol ou naturel, mettre une contrebasse dans un porte-document ou un hélicon dans un carton à chapeau" – [Dac, 1981] (this quote is also available at http://dicocitations.lemonde.fr/citations/citation-29012.php).

[27] There is a great variety of musical intonations in World Music, all with different interval values although intervals of Just fifths, principally, and fourths appear to be constant factors, though often approximate. Some ethnomusicologists claim that the octave is the predominant interval. It is predominant, indeed, but only in systems in which it is predominant by design and not by chance. For further reading: [Beyhom, 2010b ; 2017].

[28] Just intonation is a musical tuning in which the frequencies of notes are related by ratios or quantifications of small whole numbers. Any interval tuned in this way is called a Just Interval. Pure intervals correspond to the vibrational patterns found in physical objects which correlate to human perception. The two notes in any just interval are members of the same harmonic series.

[29] Ratios of string length and ratios of frequency stand in reciprocal relationship to each other: 3/4= string length and 4/3 = frequency.

[30] A generative scale is the result of a construction from which other scales are derived. The descending scale *b-a-g-f-e-d-c-b* is constructed from the alternation of fifths and fourths.

(gut of sheep, of fallow deer, of cow, of bull, etc.) and its length.

When the pitch of the central string is stable, (that is when it does not stretch any longer under a given tension) both the first string of the front and the last are tuned a just fifth away from the central string, the continuation of the process is explained in Fig. 2.

A calculation of strings parameters: length; tension; weight; section; mass, was made in order to find the most appropriate gages and tensions for stringing a lyre. I chose my 2008 replication of the silver lyre of Ur as model although it has eleven strings. I used Taylor's Equation: $T = M(2L\ F)^2$, where T is the tension of the string; M the linear mass; F is the Frequency and L the length of the string. The strings which came from the calculations were inharmonious to the organology of the lyre. They all sounded dull and could not possibly have been used some five thousand years ago, or at any time, for that lyre. I rejected them and worked with some basic "intuitive" logic: eleven twisted strands of sheep gut for bass string 11; ten strands for string 10; nine strands for string 9, eight strands for string 8, seven strands for string 7; six strands for sting 6; five strands for string 5; four strands for string 5; 4 strands for string 4; 3 strands for string 3; 2 strands for string 2 and one strand for string 1.

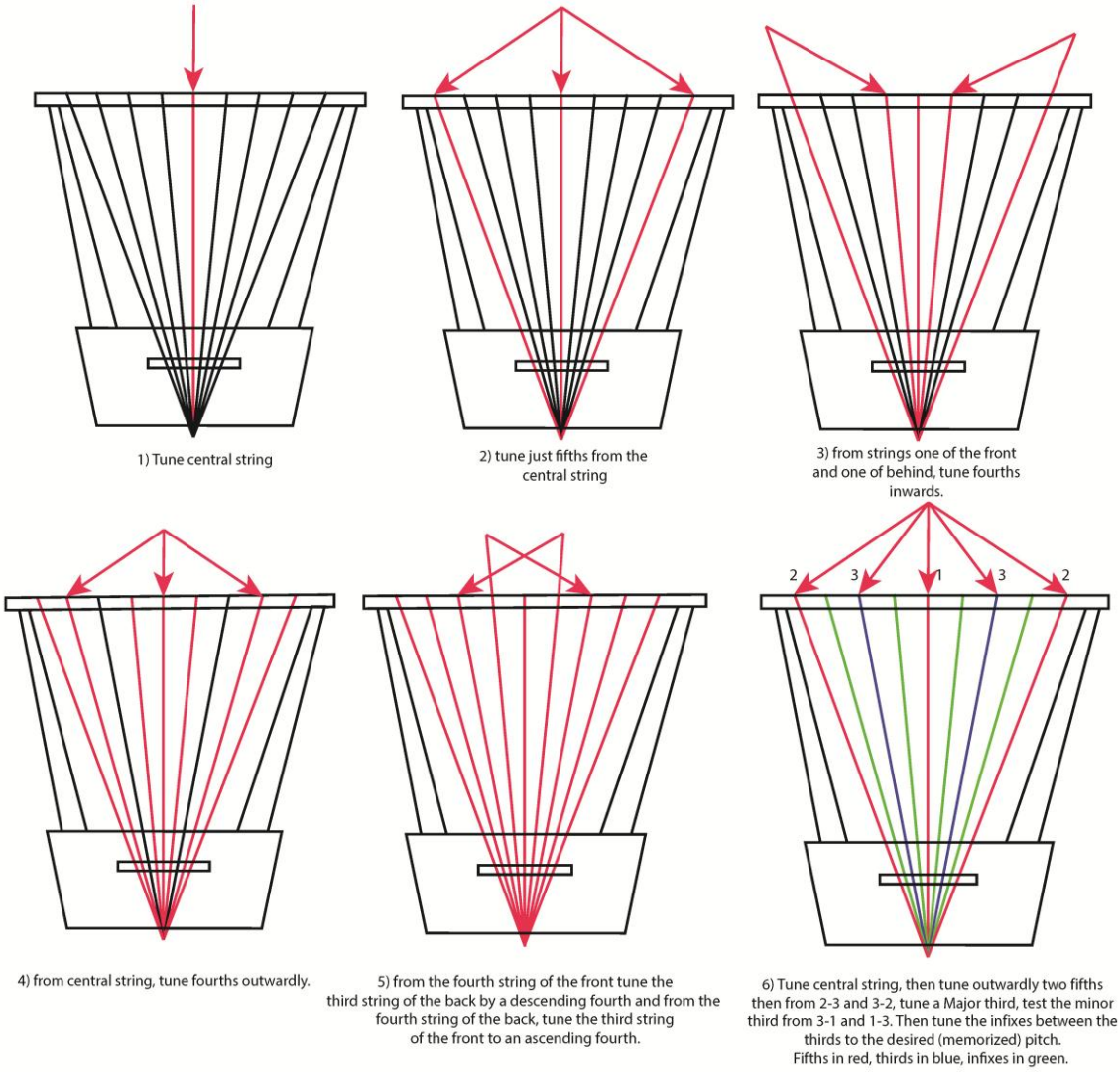


Fig. 2 (1-2-3-4-5-6). Illustrated tuning procedure derived from *nabnītu* xxxii. But it is also possible that they tuned with fifths and thirds only. In this case, after the second process when the central and two outer strings are tuned fifths apart, Just Major thirds are tuned from the outer strings. Then infixes are tuned from pitch memory. I would be in favor for this second type of tuning because it would be better suited to the interval list in the next tablet.

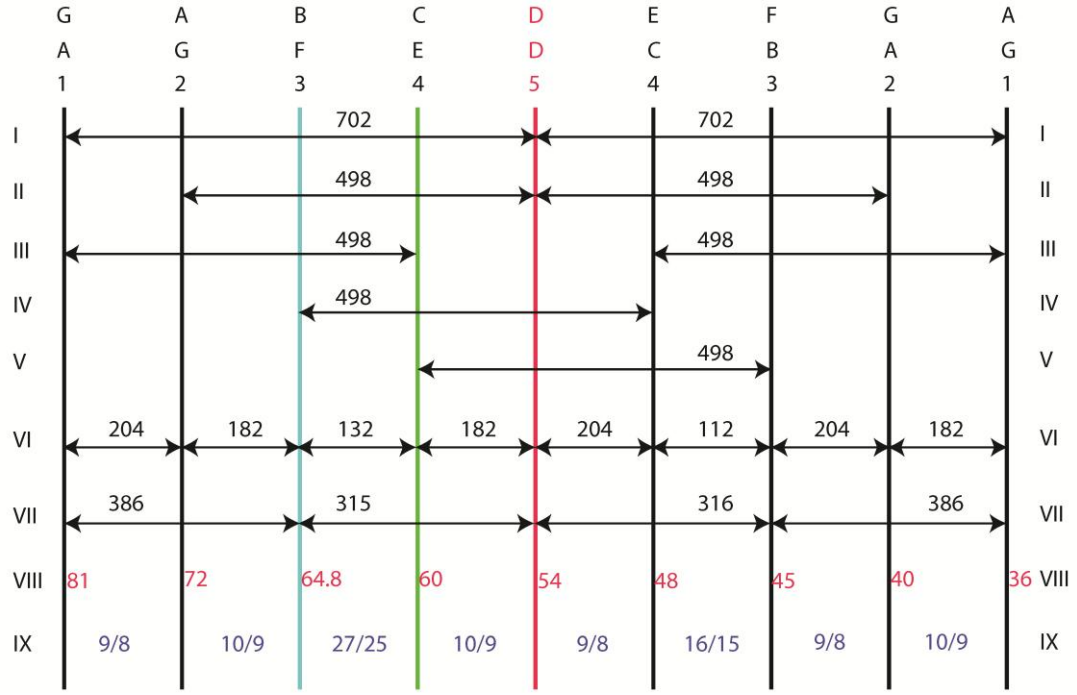


Fig. 3 Hypothetical construction of the generative Babylonian scale. I, location of fifths and value in cents; II, III, IV and V, location of fourths and size; VI, interval values of string-pitches in cents between each pitch and location; VII, interval values of major and minor thirds and location; VIII, location and pitch quantification of each string-pitch; IX, location and ratio values of each interval. Ratios are in blue; quantifications are in red.

This intuitive method proved to be the best for the lyre which now sounds at its best. Therefore, while Taylor's equation is correct for the calculation of strings for tense diatonic harps, it is totally unsuited to lyres.

The number 64.8 in Fig. 3 above (line VIII: 81; 72; 64.8; 60, etc.), which came from my hypothetic tuning in fifths and fourths, presented a problem as it needs to be multiplied by ten to become a whole regular number (64.8x10 = 648).

It could be argued that they multiplied all of their regular numbers by ten (810; 720; 648; 600, etc.) as it was done later during the Western Renaissance, and later, but I do not think they did.[31]

However, 64.8 is the value for string 3 and therefore its qualification of "third thin string" hitherto obscure, is now explicit, due to its abnormality. It is rectified with 60, the fourth string, "corrected/created" by the god Ea/ENKI, the god of music, whose qualification is thereby understood.

64.8 in relation to 45 delineates a "dissonance"[32] of 631 cents (versus 612 cents, the Just Intonation "tritone" made of three just tones of 204 cents each).

It is this "dissonance", consequence of the introduction of the semi-tone, which off-balanced an otherwise perfect pentatonic system: *a-g-e-d-c-a-g/g-a-c-d-e-g-a* which became enneadic/bi-pentadic *a-g-**F**-e-d-c-**B**-a-g/g-a-**B**-c-d-e-**F**-g-a*. All figures, included 64.8 would

[31] I believe that they did not quantify their pitches beyond 81 (in their theory) because this is the last number in the Nippur tablets, with penultimate 80. Since 81-80 = 1 (81/80=1.0125; 80/81 = 0.98765432...) and that the ratio of 81:80 = 21.506290 cents. This is the comma of Didymus, also called syntonic comma, chromatic diesis, Ptolemaic comma, or the wrongly qualified as diatonic comma, which is a small interval between two musical notes, equal to the frequency ratio 81:80 (21.51 cents). In later Greek theory, this comma is referred to as the "comma of Didymus" because it is the amount by which Didymus would have →

→ corrected the Pythagorean major third (81:64, around 407.82 cents) to a just major third (5:4, around 386.31 cents). The quantification of 81 - 80 = 1, producing the smallest interval in the Nippur list, would have ended the series, logically.

[32] The term dissonance is inappropriate. Babylonians used the terms "*la zaku*" which roughly translates as "unclear", but unclear does not mean dissonant. Therefore, although Babylonians found that interval "strange" it had not reached the qualification of "dissonant".

either have been multiplied by ten, or 64.8 would have been adjusted to 64 with the consequences that the fifth 81:54 reduced to 80:54 would have produced a smaller fifth of 680 cents as shown in Fig. 4, line VIII. I am not in favour of the second proposition.

At present, my hypothesis is all we have to speculate about Babylonian intonation. I hoped that this would be useful as basis for more punctilious research. To that end, as I was looking for mathematical cuneiform texts with series of regular numbers.

About ten years ago, I "re-discovered" four tablets found in the early 1900s at the Temple Library of Nippur.[33] They date from 2300-2200 BC. They have a series of numbers from 1 to 81. They all are regular numbers taken from the Babylonian sexagesimal system, or base 60 arithmetics, and evenly divide powers of 60. For instance, $60^2 = 3600 = 48 \times 75$, so both 48 and 75 are divisors of a power of 60. They are numbers with only prime divisors 2, 3, and 5. In music theory, the Just intonation of the tense diatonic scale involves regular numbers: the pitches in a scale have frequencies proportional to the numbers in the sequence given above from 1 to 81.[34]

Thus, for an instrument tuned in this manner, all pitches are regular numbers, therefore, harmonics of a single fundamental frequency. This scale is called a 5-limit tuning, meaning that the interval between any two pitches can be described as a product $2^i3^j5^k$ of powers of the prime numbers up to 5, or equivalently as a ratio of regular numbers.

These numbers agree with my views. They are printed in red in Fig. 3 and Fig. 4. There is no formal evidence that they were used for musical purposes. However, they end with 80 and 81. This means that the interval between them, later named by the Greek word κόμμα (*kómma*) from κόπτω (*kóptō*, "I cut"), was already known in Babylon over 4000 years ago. This strongly reinforces my opinion that the Nippur Tablets were used as the basis for pitch quantification theory, although probably not exclusively. But the question is how they could have associated these numbers with the harmonic series is difficult to understand.

The Nippur regular numbers could also have been used as practical string length standards, essential to the instrument maker who would have used them as speaking lengths of string and lengths of air columns of wind instruments. (Fig. 5)

These measurements might suggest a standard "Babylonian relative tuning".[35] They can also be read as units of frequency, but the likelihood that they understood the concept is most improbable. However, we must never underestimate Babylonian scholarship. Frequency might not have been conceptualized as we understand it, but it might have been sensed. If a string is plucked and if the tip of a finger is lightly placed at about the middle of the string, its vibrations are felt.

It is probable that they would have noted that the higher the pitch, the faster the vibrations, and reciprocally, but they would not have been able to count them. However, from their expert usage of reciprocals, they might have perceived that the reciprocals of string lengths equated to the number of their vibrations.

Conclusion

This text describes a bi-pentadic (enneadic) set. Strings gave their names to pitches. Therefore, they could have notated a melody with them although there is no evidence that they did. The Babylonian set described in this tablet is a "Just Intonation enneadic diatonic" system made up of two symmetric conjunct pentads. But it is not a heptatonic set enlarged by two degrees.

[33] [Hilprecht, 1906, p. 21, Pl. 10, 11, 12 and V].

[34] Regular numbers are numbers that evenly divide powers of 60 (or, equivalently powers of 30). As an example, $602 = 3600 = 48 \times 75$, so both 48 and 75 are divisors of a power of 60. Thus, they are regular numbers. Equivalently, they are the numbers whose only prime divisors are 2, 3, and 5. Regular numbers from 1 to 81 are: 1; 2; 3; 4; 5; 6; 8; 9; 10; 12; 15; 16; 18; 20; 24; 25; 27; 30; 32; 36; 40; 45; 48; 50; 54; 60; 64; 72; 80; 81.

[35] Relative tuning is when an instrument is tuned to itself. An absolute tuning is when the instrument is tuned to a pitch common to an orchestral, national or a 'tentative universal pitch'. See [Young, 1955], and [Lloyd and Fould, 1949, p. 84 sq.].

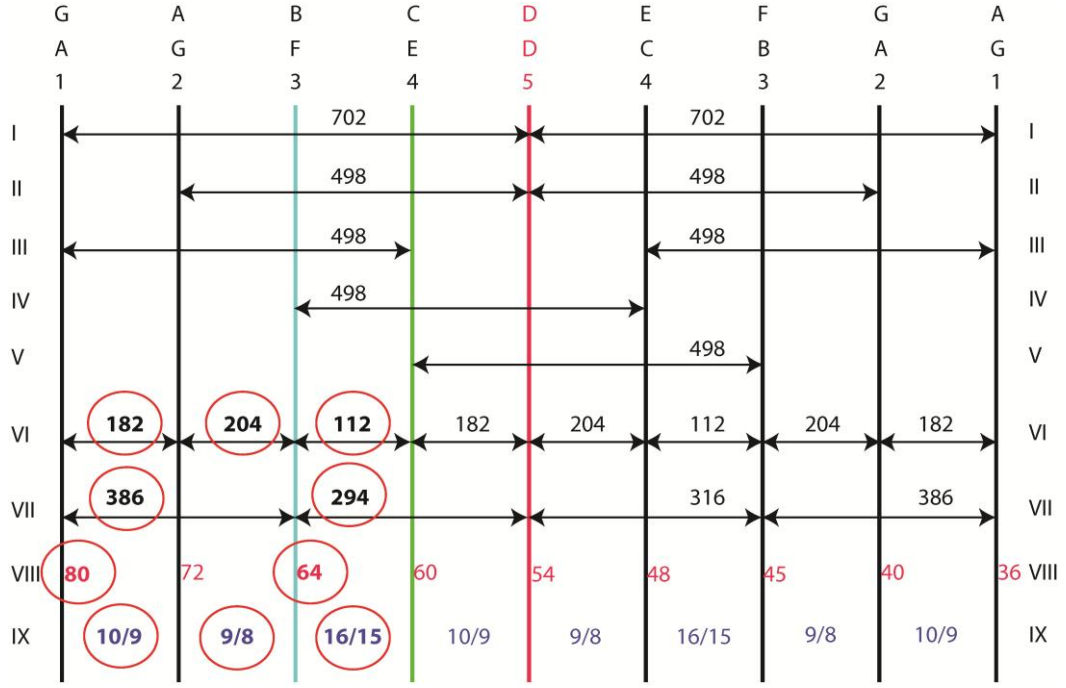


Fig. 4 Quantifications, ratios and cents in bold and underlined indicate changes due to the reformation when 64.8 was corrected to 64. This brought new quantifications in the first pentad (encircled in red). There, the tritone is 610, almost equal to 612, the tritone in Just Intonation.

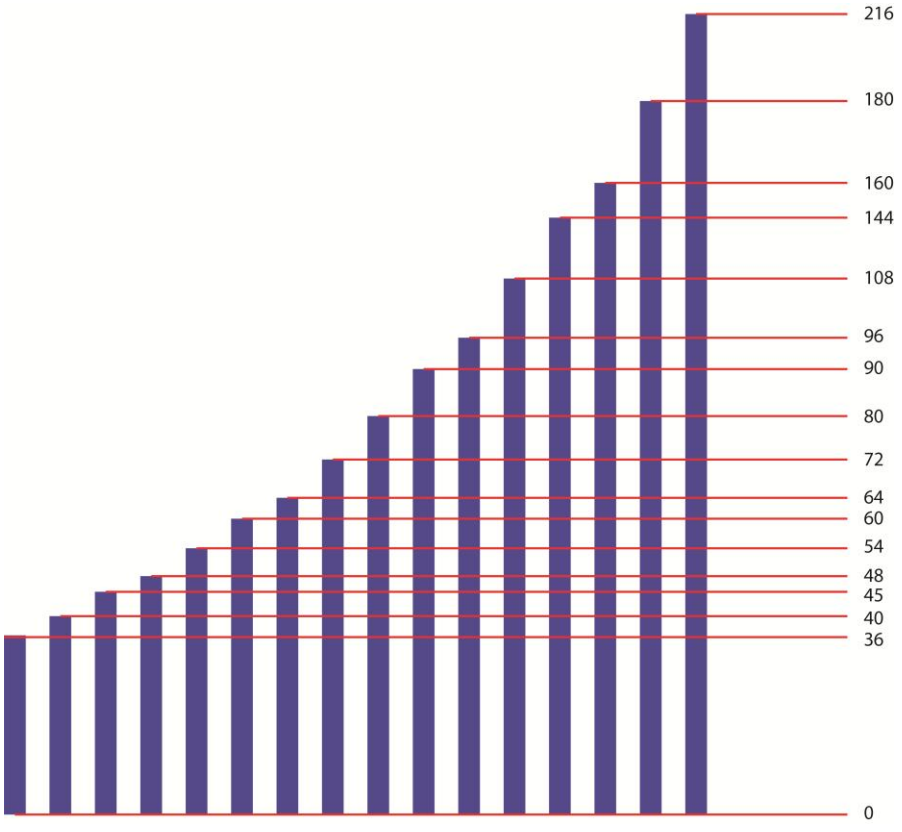


Fig. 5 Babylonian quantification as pattern for string lengths, (with identical sections and mass) or air column lengths, for instrument makers. Figures not to scale.

II - CBS 10996:[36] Occidental or Oriental?

This text is also Neo-Babylonian, perhaps a bit older than *nabnītu* xxxii. It was excavated at Nippur and first published in 1960.[37] It lists a series of names of intervals and numbers associated with them. Since numbers do not exceed seven, Kilmer and others thought that this was evidence of ascending heptatonism (Fig. 6).[38] However, it was later proven that the system is descending.[39]

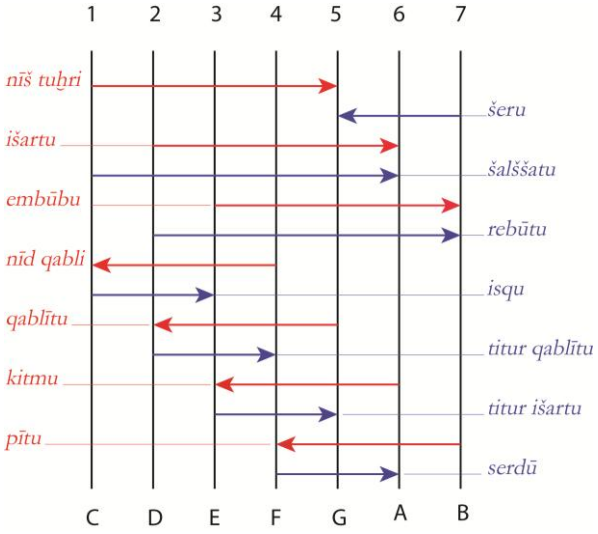


Fig. 6 Graphic representation of CBS 10996, Kilmer's version. Names in red indicate principal pentadic sets. The tetrads in this graphic are obviously inverted pentads. Names in blue are secondary (triadic, or inverted-triadic) subsets.

Fig. 7 is my graphic interpretation of the tablet. The top part (A) is Kilmer's erroneous reading of the text. It is, she claims, a pattern spanning seven steps numbered from the bottom, suggesting that "1" is the lowest pitch, and that therefore the structure is ascending. The bottom part (B) shows my reconstruction of an original and hypothetical tablet, forerunner of CBS 10996. It displays a regular pattern spanning thirteen steps starting with number "1" at the top, suggesting that "1" is the highest pitch. For the sake of clarity, let us agree that "1" = "c".

At the first line (I) of top part (A), Kilmer"s interpretation, 1↑5 (*nīš tuḫri*) is *c*↑*g*. (since Kilmer sees it ascending).

At the first line (I) of the bottom part (B) of my reconstruction, 1→5 = *c*→*f* is descending.

Line II in (A) is descending 7→5 = *b*→*g* with Kilmer.

Line II in (B) is ascending 7↑5 (*šeru*). It is "*d*↑*f*".

The rest of my graphic representation where part (B) is the reconstruction which would have been the triskaidecadic[40] source for part (A). The erratic arrangement of part (A), the original CBS 10996, was left unexplained even as recently as 2013.[41]

Obviously, the arrhythmic order of the intervals in CBS 10996 is the consequence of the adaptation of a larger system into a smaller one, of seven pitches, or for an instrument fitted with seven strings.

The scribe who wrote the text kept the original polarities[42] of the intervals as they were in the original text, in his adaptation. It explains the inconsistencies in the numbers. Obviously, the scribe knew about the larger span which he was adapting, (Fig. 8 and Fig. 4) probably as an exercise, to an instrument with seven string/pitches.

Such an instrument would have been designed exclusively for music composed from a heptatonic system, obviously.

[36] [Gurney, 1974, v. VII (1973), Pl. 126].

[37] [Kilmer, 1960]. In her paper, Kilmer does not write anything worth mentioning about music. In another article entitled "The Strings of Musical Instruments: their Names, Numbers and significance", [Kilmer, 1965], she makes interesting philological remarks but no progress with musicology. The article has an appendix written by Duchesne-Guillemin who wrongly confirms that the scale is ascending. Another article by David Wulstan, "The Earliest Musical Notation", [Wulstan, 1971], is also misguided. Another paper by Kilmer, "The Discovery of an Ancient Mesopotamian Theory of Music", [Kilmer, 1971], confirms that she has concocted a whole theory resting on the flawed interpretation of one text only.

[38] A scale of seven degrees as: *c-d-e-f-g-a-b*; *d-e-f-g-a-b-c*; *e-f-g-a-b-c-d*, etc.

[39] The rising or falling of a system is only relevant to the theoretical process but is irrelevant to praxis.

[40] From Ancient Greek τρεισκαίδεκα (*treiskaídeka*, "thirteen"), from τρεῖς (*treîs*, "three") + καί (*kaí*, "and") + δέκα (*déka*, "ten"). Hence triskaidecadic, adj.

[41] [Mirelman, 2013, p. 46, fn. 6]: "The order in which the dichord pairs are referred to here (e.g., '5-2' as opposed to '2-5') corresponds to the order in which they occur in the theory texts. The theory texts enumerate the dichords according to a pattern that is not consistently ascending or descending."

[42] The polarity of an interval is defined by which note comes first: c↘f (1↘5) suggests that c (1) is first played, followed by lower f (5). In CBS 10996, polarity is given in number and pitch order.

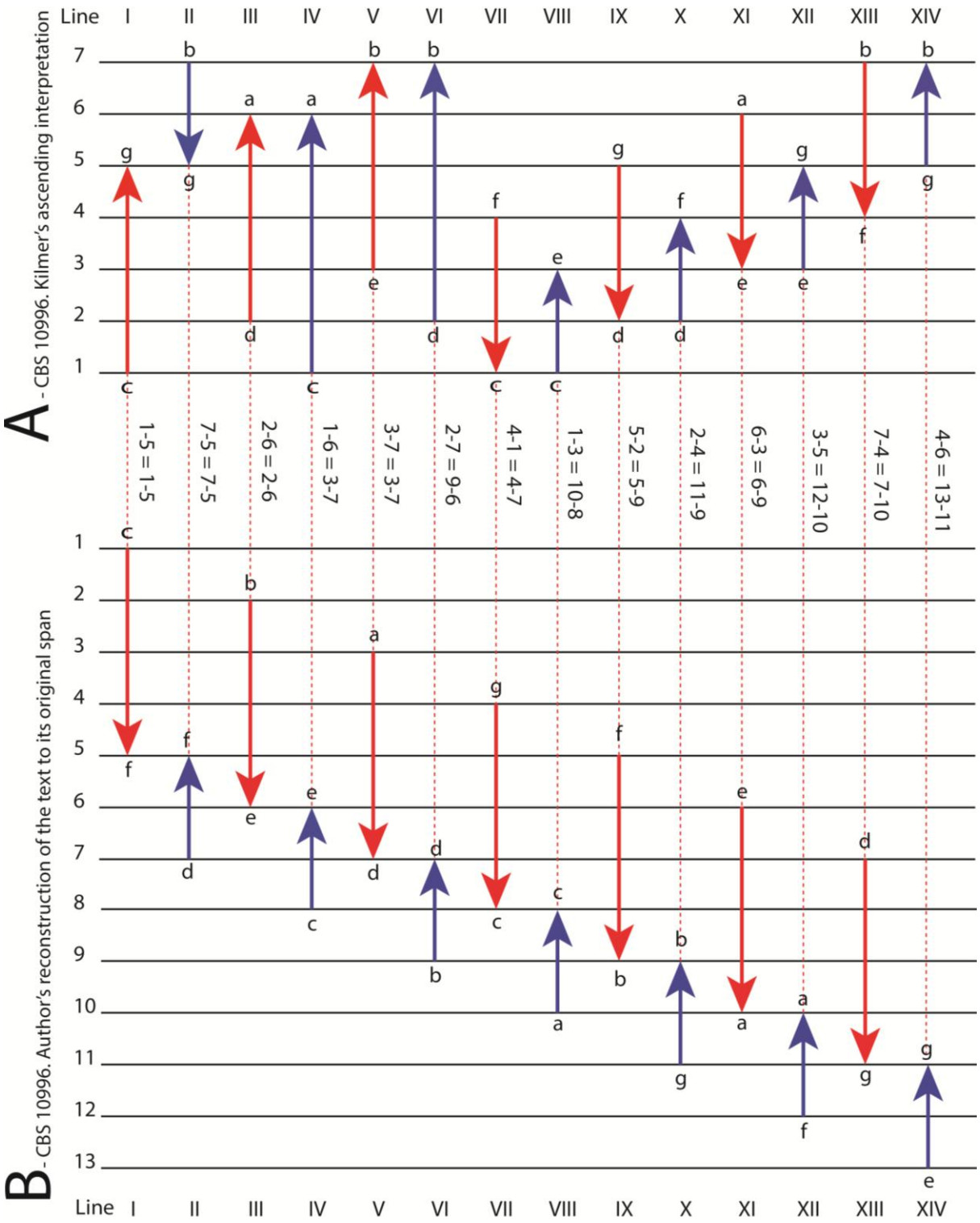


Fig. 7 The seven lines at the top are the transliteration of the original text in its descending polarity. Lines 1 to 13 of the lower graphic shows the reconstruction of what would have been the original span, in a descending order. Numbers between lines 1 of A and 1 of B give interval inversions when they appear.

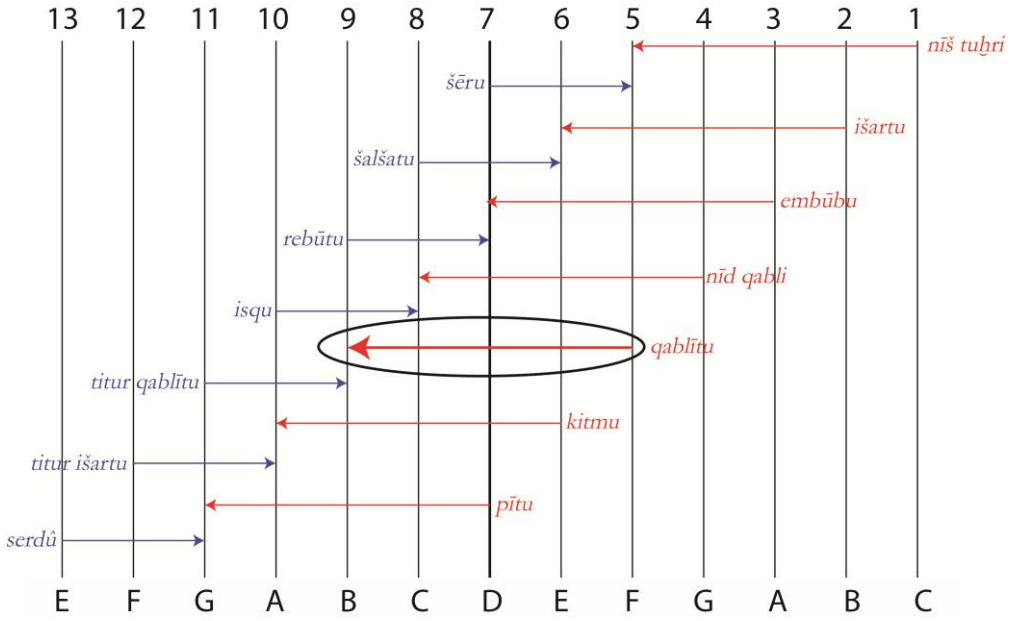


Fig. 8 Author's reconstruction of the Greater Babylonian System of descending pentads and ascending triads. The position of Babylonian pentads is extrapolated from text CBS 10996. Therefore, this is how they would have been located prior to their contraction into a heptatonic framework. I have inscribed *qablītu* in an ellipse because in Babylonian the term means 'middle' which in this case is perfectly suited. *qablītu* sits exactly in the middle of the grid, it is the only tritonic pentad in the system and is symmetrical with *D* as its axis: $B_{(semitone)}C_{(Tone)}D_{(Tone)}E_{(semitone)}F$.

But I would like to be clear in my opinion that it was certainly not a catalogue of intervals that musicians would have used for writing compositions or playing pieces.

The two most puzzling questions are, firstly, why eminently intelligent Babylonian theoreticians could have devised such an incredibly ill-conceived method? In any literate and illiterate culture[43] in the world, past and present, music is notated, or memorized from successions of pitches. That Babylonians would have been restricted to compose with dyads is inconceivable. Intervals of dyads cannot be used for melodic notation since the human voice can only sing one pitch at a time;[44] and secondly, why eminently intelligent contemporary scholars could not have seen that their interpretation of the Babylonian notation of melody was flawed,[45] is impossible to understand.

But what is even more difficult to understand is that while early Greek theory, which is built up from the same basic principles,[46] is undisputed, its Babylonian origins are deemed whimsical.

The numbers printed and encircled in red in Fig. 4 are speculative. The ennead or "nine pitch system" is perfectly symmetric in a Just Intonation construction made from the alternation of just fifths and just fourths.

[43] By literate I mean cultures which are musically literate/numerate. Musical literacy/numeracy is certainly not essential to music theory and practice. Oral usage of contiguous pitches is not the prerogative of the literate. Music existed a long time before the written language and it is obvious that the earliest attempts at writing down theory rested on orality.

[44] However, there is a form of "polyphonic overtone singing" by which the singer can produce overtones, one at a time above the fundamental pitch, as well as undertones. This is known in various cultures such as in Inner-Mongolia, Tibet, etc. It is also called "harmonic singing". But this technique would not allow to sing two unrelated pitches at a time as in Kilmer's hypothesis. YouTube →
has many examples of this polyphonic overtone singing. See [Anon. "Overtone singing", 2017].

[45] [Hagel, 2005]. This article must be read *in extenso* to judge for oneself the ways by which Hellenistic supremacists attempt at segregating knowingly and deviously Greek from Oriental theory and praxis, in order to majorize the one and pejorize the other, respectively. This is done despite the evidence under the form of cuneiform texts which, to the contrary, proves that it was the Greeks who "borrowed" all they could from Babylonian scholarship. It would be laborious to list these cuneiform texts but the essential ones are studied in the present paper.

[46] [West, 1994, p. 219–223].

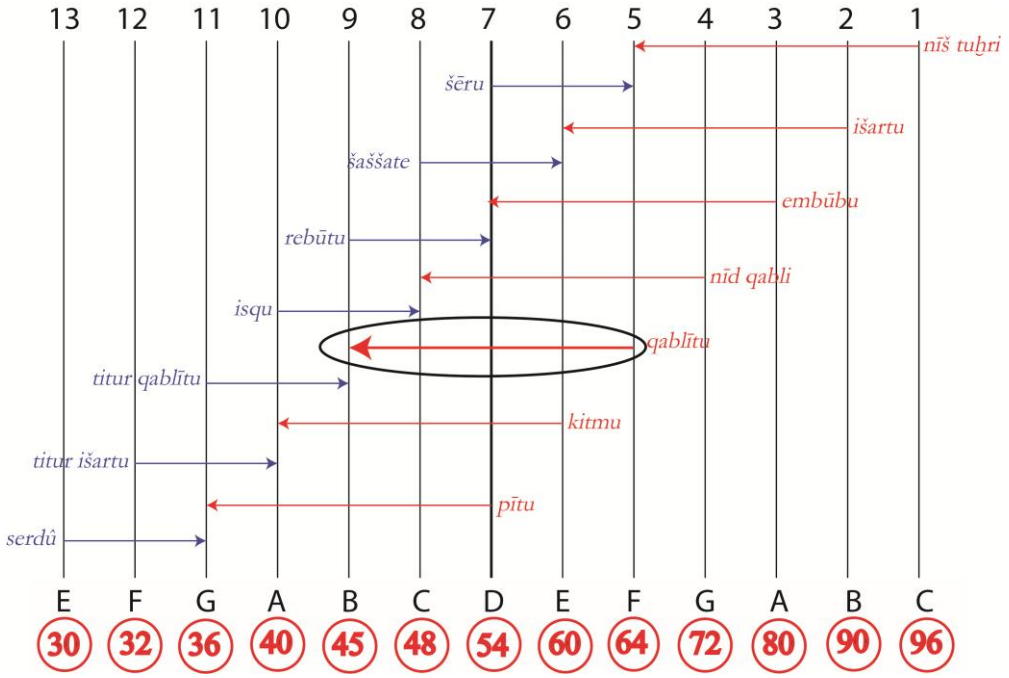


Fig. 9 Superimposition of quantifications to the original interval series. Quantification 64.8 has been reduced to 64.

My reconstruction of CBS10996 in its expanded version in Fig. 8 and Fig. 4 assumes that the central pitch in the interval *qablītu*, is the axis of symmetry for the whole system, in its original status.[47] This interval is made up of (27/25 [great limma] = 133.237 cents) + (10/9 [minor tone of just intonation] = 182.40 cents) + (9/8 [major tone] = 203.91 cents) + (16/15 [just semi-tone] = 111.73 cents) = 631.28 cents, which is an approximate acute diminished fifth.

In case of the adjustment of 64.8 to 64, the intervals of which *qablītu* is made are (16/15 [just semi-tone] = 111.73 cents) + (10/9 [minor tone of just intonation] = 182.40 cents) + (9/8 [major tone] = 203.91 cents) + (16/15 [just semi-tone] = 111.73 cents) = 609.78 cents, which is an approximate diminished fifth.

Both values which would be the Babylonian counterpart of the later Greek tritone are made up of the four different intervals with which the ennead is built and not of three just tones in the Greek system.

However, neither form of *qablītu* is tritonic as both are made up of four intervals. It is conceivable that both forms were considered as dissonant. This would explain the Babylonians term *la zaku,* meaning "unclear", *i.e.* "dissonant"[48] but it is not possible to determine its exact value as it was, as we shall see in the next text, based on tension and relaxation. I suggest that in Babylon the concept of dissonance was not clearly defined, or rather was not confined to a specific interval. In Babylonian theory, there is no known term for other forms of dissonance.

The aforementioned mathematical texts from the Nippur Temple Library have series of numbers from 1 to 81. They are all regular numbers taken from the Babylonian sexagesimal system, or base 60 arithmetics and evenly divide powers of 60.[49]

The Nippur numbers agree with my hypothesis. They are the numbers with only prime divisors 2, 3, and 5. In music theory, the Just intonation of the "diatonic" scale involves regular numbers: the pitches in a scale have frequencies proportional to the numbers in the sequence given above from (in our case) 36 to 81 or to 80.

81 is the first pitch of the generative descending scale (as derived from *nabnītu* xxxii). It is preceded by 80.

[47] "Original status" is the series of pitches resulting from a generative construction, in this case, as given in *nabnītu* xxxii.

[48] See also fn. 32, p. 97.

[49] For instance, $60^2 = 3600 = 48 \times 75$, so both 48 and 75 are divisors of a power of 60.

The ratio of 81:80 known as the aforementioned *comma* of Didymus,[50] which is of great importance in musicology, was already known at Babylon.

To conclude, the manner in which this scale is symmetrically built as 1-2-3-4-5 and 5-4-3-2-1, means that it was made up of two conjunct pentads, *i.e.* two pentads sharing a common pitch "D". Therefore, "two conjunct pentads" is a better description of what I call "enneadic set". This survived in Greece where "two conjunct tetrads" is a better description of hepta-tonism.

III - U.7/80 = UET VII, 74: *Not a tuning text!*

This text is further evidence, among many others, of the remarkable creative genius of Babylonian scholarship. The method explained in this system was never equalled in any other civilization, as it translates a dynamic disposition into its thetical[51] form.

This third tablet dates from the Old-Babylonian period, about 1800 BC. It was unearthed by Sir Leonard Woolley at Ur[52] and was published about forty years later, in 1968, by Professor Gurney.[53] At that time no one had yet hypothesised that the scale might be descending. Despite my attempts at promoting the idea on the basis of Greek and Oriental models, I was ignored. Consequently, and despite having asked advice from Oxford musicologist David Wulstan, Gurney's paper was published with the premise that the system was ascending.

Then in 1982, the Syrian Raoul Gregory Vitale[54] also attempted at promoting a descending system but was likewise ignored. At last, in 1990, Assyriologist friend Th.J.H. Krispijn perceived a new reading of line 12 as *nu-su-ḫ[um*, a form of the verb *nasāḫum*, "to tighten". This new term *nasāḫum*, Sumerian gíd-i, or *nussuḫum*, Sumerian zi-zi, is the technical verb for "to tighten" strings. Its antonym is *ne'ûm*, Sumerian tu-lu. Subsequently Gurney published another paper in 1994[55] where it was finally established that the Babylonian system was descending[56] on the basis that the strings must be tightened in part one of the text while it was assumed that strings were to be loosened in the previous publication of 1968. So, it was philology which won the case for musicology: Assyriologists did not trust musicologists.

The Text: restoration, translation and commentaries

Transliteration of U.7/80

[*šum-ma* ^{giš}ZÀ.MÍ *pi-i-tum-ma*]
1 [*e-e*]*m-b*[*u-bu-um la za-ku*]
2 *ša-al-š*[*a-am qa-at-na-am tu-na-sà-aḫ-ma*]
3 *e-em bu-bu-u*[*m iz-za-ku*]

4 *šum-ma* ^{giš}Z]À.MÍ *e-em-bu-bu-um-ma*]
5 *ki-it-mu-um* [*la za-ku*]
6 *re-bi úḫ-ri-im* [*tu-na-sà-aḫ-ma*]
7 *ki-it-mu-um i*[*z-za-ku*]

8 *šum-ma* ^{giš}ZÀ.MÍ *k*[*i-it-mu-um-ma*]
9 *i-šar-tum la za-*[*ka-at*]
10 *ša-mu-ša-am ù-úḫ-ri-a-a*[*m tu-na-sà-aḫ-ma*]
11 *i-šar-tum iz-za-*[*ku*]

12 *nu-su-ḫ*[*u-um*]

13 *šum-ma* ^{giš}Z]À.MÍ *i-šar-t*[*um-ma*]
14 *qa-ab-li-ta-am ta-al-pu-*[*ut*]
15 *ša-mu-ša-am ù-úḫ-ri-a-am te-*[*ni-e-ma*]
16 ^{giš}ZÀ.MÍ *ki-it-mu-*[*um*]
17 [*šum*]*-ma* ^{giš}ZÀ.MÍ *ki-it-m*[*u-um-ma*]
18 [*i-ša*]*r-ta-am la za-ku-ta-am t*[*a-al-pu-ut*]
19 [*re-bi*] *úḫ-ri-im te-ni-e*/[*-ma*]
20 ^{giš} ZÀ.MÍ *e-em-bu-bu-um*]

Certain words in this transliteration have a final mimation, an 'm' following the case ending of a word, *i.e. išartum* instead of *išartu*. This practice is typical of the Old-Babylonian period.

[50] See fn. 31, p. 97.

[51] Thetic means "set down or stated positively or absolutely". From Greek "*thetikos*" = that can be placed < "*tithenei*" = to place. It describes sets translated from the dynamic layout in a disposition as is described with text U.7/80. It is another manner in which to notate a scale, from Greek "*dynamikos*", "powerful". It describes the layout of pitches as in the GBS (Greater Babylonian System). Dynamic *a-b-c-d-e-f-g-a* is thetic *c-d-e$^{\flat}$-f-g-a$^{\flat}$-b-c*, in case the thetic is set on the scale of "*c*" with accidentals added according to a given dynamic scale.

[52] [Gurney, 1974], Pl. 74.

[53] [Gurney, 1968].

[54] [Vitale, 1982].

[55] [Gurney, 1994].

[56] However, a descending system is not appropriate for lute types where the strings are "stopped". They require an ascending system because the system starts by an open string. Complementary pitches are produced by the position of finger tips along the neck of the instrument. This results in an ascending system.

It was on this basis that Gurney translated the text, and reconstructed it partially by extrapolation as follows:

"First part
1. If[57] the harp is *išartum*
the 'unclear interval' between strings 5 and 2 is *qablītum* (should be 5-1^{b})
tighten by a 'semi-tone'[58] string 5
the harp will be *qablītum*"

This first quatrain of the first part was reconstructed by Professor Gurney. My interpretation is that the set of *išartum* comes from the conjunction of pentads *nīš tuḫrim* and *qablītum* = *c-b-a-g-f* + *f-e-d-c-b* as explained in my reconstruction of CBS 10996. But the "unclear interval" is not between strings "5" and "2 of the front" (although in theory it exists as an "unclear" fourth at that position) but between "5" (*ḫa-am-šu*) and the "1 behind string" (*úḫ-ru-um*), an "unclear" fifth.

Now, that it was strings "5" and "2 of the front" which located the "unclear interval" would not have been written as such in the original text. It would have said that *qablītum* is *la zaku* which means, as mentioned before, "unclear", *i.e.* unpleasant.[59] The substitution of string "1 of the behind" to "2 of the front" is the consequence of the erroneous reading of CBS 10996, and is used to suggest heptatonism.

"2. If the harp is *qablītum*
the 'unclear interval' between strings 1 and 5 is *nīš tuḫrim* (correct)
tighten strings 1 and 8 (should be 1^{f}-2^{b})
the harp will be *nīš tuḫrim*"

The set of *qablītum* comes from the conjunction of pentads *qablītum* and *išartum* = *f-e-d-c-b* + *b-a-g-f-e*. Therefore, the "unclear interval" is between strings "1 of the front" and "5". The reconstruction says that strings 1 and 8 should be tuned-up by a "semi-tone". But it should be written that it is string 1 of the front and string 2 of the back which should be tuned-up.

"3. If the harp is *nīš tuḫrim*
the 'unclear interval' between strings 4 and 1 is *nīd qablim* (should be 4^{f}-2^{b})
tighten string 4
the harp will be *nīd qablim*"

The set of *nīš tuḫrim* comes from the conjunction of pentads *išartum* and *kitmum* = *b-a-g-f-e* + *e-d-c-b-a*. The reconstructed text says that the "unclear interval" is between strings "4 of the front" and "1 of the front". Here again, it should be placed between strings "4 of the front" and string "2 of the back".

"4. If the harp is *nīd qablim*
the 'unclear interval' between strings 7 and 4 is *pītum* (should be 7-11)
tighten string 7
the harp will be *pītum*"

The set *nīd qablim* comes from the conjunction of pentads *kitmum* and *embūbum* = *e-d-c-b-a* + *a-g-f-e-d*. Here, the limitation of the span for the set places the "unclear interval" *pītum* between strings "3 of the behind" and "4 of the front" and it is string "3 of the behind" which must be tuned-up.

"5. If the harp is *pītum*
the 'unclear interval' between strings 3 and 7 is *embūbum* (should be 3^{f}-3^{b})
tighten string 3
the harp will be *embūbum*"

The set of *pītum* comes from the conjunction of pentads *embūbum* and *pītum* = *a-g-f-e-d* + *d-c-b-a-g*. The transliteration of the tablet, since this is where the text U.7/80 starts, says that the "unclear interval" *embūbum* is placed between strings "3 of the front" and string "3 of the behind" and that string "3 of the front" should be tuned-up.

"6. If the harp is *embūbum*
the 'unclear interval' between strings 6 and 3 is *kitmum* (should be 6-10)
tighten string 6
then the harp will be *kitmum*"

[57] The Akkadian term *šumma* has been consistently translated by "when" although it should be the conditional "if". It was argued that it meant the same thing. The Babylonians were keen on the usage of *protasis* (if) and *apodosis* (then): a protasis is the clause expressing the condition in a conditional sentence (e.g. *if you asked me* in *if you asked me I would agree*). The apodosis is the main clause of a conditional sentence (*e.g. I would agree* in *if you asked me I would agree*).

[58] The text does not say "semi-tone". It is an amount by which the "unclear interval" is corrected. This quantity is unknown as the system does not rely on ratios and therefore is left to the appreciation of the musician's tonal memory. I shall replace the term by "tighten".

[59] See also fn. 32, p. 97.

The set of *embūbum* comes from the conjunction of pentads *pītum* and *nīd qablim* = *d-c-b-a-g* + *g-f-e-d-c*. The "unclear interval" is *kitmum*. It should be placed on strings 9-10 of the Greater System.

"7. If the harp is *kitmum*
the 'unclear interval' between strings 2 and 6 is *išartum* (should be 2^{f}-4^{b})
tighten strings 2 and 9
the harp will be *išartum*"

The set of *kitmum* comes from the conjunction of *nīd qablim* and *nīš tuḫrim* = *g-f-e-d-c* + *c-b-a-g-f*. The "unclear interval" *išartum* is located between strings 2 and 6, while it should be string "2 of the front" and string "4 of the back".

The second part is the reverse of the first part.

Musical Quantification of U.7/80

Although incomplete, this fragmentary text holds a wealth of knowledge which coincides with the information extracted from the previous texts, *nabnītu* xxxii and CBS 10996.

The method given in U.7/80 places seven sets on a bi-pentadic span, or on an instrument with nine strings by simple tuning of one or two of its strings. This gives the following enneadic sets:

Part 1

išartum (1):	*c-b-a-g-f-e-d-c-b*
qablītum:	*c-b-a-c-$f^{\#}$-e-d-c-b*
nīš tuḫrim:	*$c^{\#}$-b-a-g-$f^{\#}$-e-d-c-b*
nīd qablim:	*$c^{\#}$-b-a-$g^{\#}$-$f^{\#}$-e-d-$c^{\#}$-b*
pītum:	*$c^{\#}$-b-a-$g^{\#}$-$f^{\#}$-e-$d^{\#}$-$c^{\#}$-b*
embūbum:	*$c^{\#}$-b-$a^{\#}$-$g^{\#}$-$f^{\#}$-e-$d^{\#}$-$c^{\#}$-b*
kitmum:	*$c^{\#}$-b-$a^{\#}$-$g^{\#}$-$f^{\#}$-$e^{\#}$-$d^{\#}$-$c^{\#}$-b*
išartum (2):	*$c^{\#}$-$b^{\#}$-$a^{\#}$-$g^{\#}$-$f^{\#}$-$e^{\#}$-$d^{\#}$-$c^{\#}$-$b^{\#}$*

12 *nu-su-ḫ[u-um]*

Part 2

išartum (2):	*$c^{\#}$-$b^{\#}$-$a^{\#}$-$g^{\#}$-$f^{\#}$-$e^{\#}$-$d^{\#}$-$c^{\#}$-$b^{\#}$*
kitmum:	*$c^{\#}$-b-$a^{\#}$-$g^{\#}$-$f^{\#}$-$e^{\#}$-$d^{\#}$-$c^{\#}$-b*
embūbum:	*$c^{\#}$-b-$a^{\#}$-$g^{\#}$-$f^{\#}$-e-$d^{\#}$-$c^{\#}$-b*
pītum:	*$c^{\#}$-b-a-$g^{\#}$-$f^{\#}$-e-$d^{\#}$-$c^{\#}$-b*
nīd qablim:	*$c^{\#}$-b-a-$g^{\#}$-$f^{\#}$-e-d-$c^{\#}$-b*
nīš tuḫrim:	*$c^{\#}$-b-a-g-$f^{\#}$-e-d-c-b*
qablītum:	*c-b-a-c-$f^{\#}$-e-d-c-b*
išartum (1):	*c-b-a-g-f-e-d-c-b*

The strings which are tuned up are printed red. Note that the last *išartum* is not at the octave of the first *išartum*. It is a "semi-tone" (of an undetermined value) higher. Therefore *išartum* (1) is not equal to *išartum* (2).

The indications in the text are "to tighten" and "to loosen" the strings. The quantity by which it should be is not given. This means that while theoretically we should have a Just Intonation system, in practice it might have been quite different in function of mood and other factors, such as location, time of day, season, but also and most importantly on tonal memory. Had they insisted on precise pitches, they would have indicated them in ratios with which they were fully conversant.

However, ratios are meaningless on harps or lyres, and this is probably why they were not used. They are only effective when a string is divided with frets or fret-marks as guides, on lute types.

For the sake of demonstration, should we hypothesize that this structure was intended for Just Intonation, then Fig. 10 gives such quantifications.

The recital in U.7/80 is an exceptional narrative for the history and transmission of the earliest musical construction, from its pre-literate form onward. Then, with the advent of literacy, musicology over-flowed its banks to grow into the most sophisticated form ever achieved in any civilization – four thousand years ago. The reliability of the Sumero-Babylonian scribal discipline was such that even with the few tablets which have reached us by luck – and not by design – it was nevertheless possible to decipher some of the unique intricacies of Mesopotamian music making.

Text CBS 10996 describes the reduction to a heptachord of a triskaidecadic forerunner. It suggests a foundation pitch around which other pitches agglutinate in a manner not dissimilar to the development of language. They agglutinate as sets and subsets, pentadic and triadic, respectively.

Two conjunct triads (Fig. 11) make a pentad: *serdû* +*titur qablītu* = *išartu* (*E-F-G/G-A-B* rising); *titur išartu* + *isqu* = *niš tuḫri* (*F-G-A/A-B-C* rising); *titur qablītu* + *rebūtu* = *pītu* (*G-A-B/B-C-D* rising); *isqu* + *šalšatu* = *kitmu* (*A-B-C/C-D-E* rising); *rebūtu* + *šeru* = *qablītu* (B-C-D/D-E-F rising).

Tune up									
	I	II	III	IV	V	IV	III	II	I
išartum	1290	1200	996	792	588	498	294	90	0
qablītum	1290	1200	996	792	680	498	294	90	0
nīš tuḫrim	1382	1200	996	792	680	498	294	182	0
nīd qablim	1382	1200	996	884	680	498	294	182	0
pītum	1382	1200	996	884	680	498	386	182	0
embūbum	1382	1200	1088	884	680	498	386	182	0
kitmum	1382	1200	1088	884	680	590	386	182	0
išartum	1382	1292	1088	884	680	590	386	182	92
Tune down									
išartum	1382	1292	1088	884	680	590	386	182	92
kitmum	1382	1200	1088	884	680	590	386	182	0
embūbum	1382	1200	1088	884	680	498	386	182	0
pītum	1382	1200	996	884	680	498	386	182	0
nīd qablim	1382	1200	996	884	680	498	294	182	0
nīš tuḫrim	1382	1200	996	792	680	498	294	182	0
qablītum	1290	1200	996	792	680	498	294	90	0
išartum	1290	1200	996	792	588	498	294	90	0

Fig. 10 Analysis of values of sets in cents showing that the last *išartum* of the first chapter is not at the octave of the first set. The same applies to the second chapter. The two sets differ by 92 cents throughout. 92 cents is the larger *limma* which is the defect of a fourth, 498 cents, increased by an *apotome*, 112 cents (total 610 cents) from a fifth, 702 cents, and hence the interval by which the fourth must be sharpened to be an *apotome* below (*i.e.* the 'leading note' to) the fifth and hence the interval by which the fourth is sharpened on modulating into the dominant.

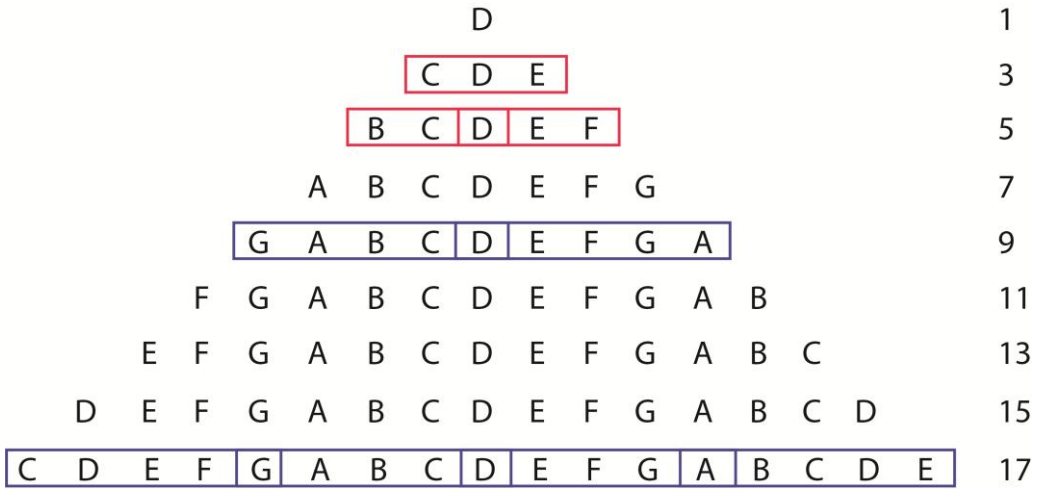


Fig. 11 Pyramid of systems. This pyramid shows the gradual structure from an initial pitch, to a triad, then the conjunction of two triads amounting to a pentad, then of two conjunct pentads amounting to an ennead, then of four conjunct pentads amounting to the Greater Babylonian System or heptadecade.

The reason for their position in my reconstruction of the Greater Babylonian System is not yet understood. However, their integration in my interpretation of the Hurrian song H6 corroborates their presence, complementing descending fifth, logically and aesthetically.

As they stand, these triads are an essential part of the Babylonian scape-sound. The Greater Babylonian System spreads onto 17 pitches with smaller spans of 15, 13, 11, 9, 7, 5 and 3, (Fig. 11) all based on the same principle of the sharing of a common axis of symmetry. This is supported by the iconography where the number of strings vary with periods coinciding with organological trends.

Pitch sets are composed of two conjunct pentads where the last pentad of a given ennead is also the first pentad of the ennead which follows (Fig. 12, Fig. 13 and Fig. 14):

- Pentads *nīš tuḫri* + *qablītu* = set of *išartu* = ***c-b-a-g-f-e-d-c-b***
- Pentads *qablītu* + *išartu* = set of *qablītu* = ***f-e-d-c-b-a-g-f-e***
- Pentads *išartu* + *kitmu* = set of *nīš tuḫri* = ***b-a-g-f-e-d-c-b-a***
- Pentads *kitmu* +*embūbu* = set of *nīd qabli* = ***e-d-c-b-a-g-f-e-d***
- Pentads *embūbu* + *pītu* = set of *pītu* = ***a-g-f-e-d-c-b-a-g***
- Pentads *pītu* + *nīd qabli* = set of *embūbu* =***d-c-b-a-g-f-e-d-c***
- Pentads *nīd qabli* + *nīš tuḫri* = set of *kitmu* = ***g-f-e-d-c-b-a-g-f***

or are composed of two conjunct pentads which also follow each other, conjunctly, where the last pitch of a set is the first pitch of the next and results in an order of descending contiguous pitches:

- Pentads *nīš tuḫri* + *qablītu* = set of *išartu* = ***c-b-a-g-f-e-d-c-b***
- Pentads *išartu* + *kitmu* = set of *nīš tuḫri* = ***b-a-g-f-e-d-c-b-a***
- Pentads *embūbu* + *pītu* = set of *pītu* = ***a-g-f-e-d-c-b-a-g***
- Pentads *nīd qabli* + *nīš tuḫri* = set of *kitmu* = ***g-f-e-d-c-b-a-g-f***
- Pentads *qablītu* + *išartu* = set of *qablītu* = ***f-e-d-c-b-a-g-f-e***
- Pentads *kitmu* +*embūbu* = set of *nīd qabli* = ***e-d-c-b-a-g-f-e-d***
- Pentads *pītu* + *nīd qabli* = set of *embūbu* = ***d-c-b-a-g-f-e-d-c***

Unequivocally, this system, whether of 17, 15, 13, 11, 9, 7, 5 or 3 pitches, is built from pentads and triads and can also integrate a set of seven pitches.

The "unclear intervals" are located at the following positions:

- In *išartu*, ***c-b-a-g-f-e-d-c-b***=1-2-3-4-5-4-3-2-1, the "unclear interval" is *qablītu* and is placed on 5-1b = ***f-b***
- In *qablītu*, ***f-e-d-c-b-a-g-f-e***=1-2-3-4-5-4-3-2-1, the "unclear interval" is *nīš tuḫri* and is placed on 1f-5 = ***f-b***
- In *nīš tuḫri*, ***b-a-g-f-e-d-c-b-a***=1-2-3-4-5-4-3-2-1, the "unclear interval" is *nīd qabli* and is placed on 4f-2b = ***f-b***
- In *nīd qabli*, ***e-d-c-b-a-g-f-e-d***=1-2-3-4-5-4-3-2-1, the "unclear interval" is *pītu* and is placed on 3b-4f (7-11) = ***f-b***
- In *pītu*, ***a-g-f-e-d-c-b-a-g***=1-2-3-4-5-4-3-2-1, the "unclear interval" is *embūbu* and is placed on 3f-3b = ***f-b***
- In *embūbu*, ***d-c-b-a-g-f-e-d-c***=1-2-3-4-5-4-3-2-1, the "unclear interval" is *kitmu* and is placed on 4b-3f (6-10) = ***f-g***
- In *kitmu*, ***g-f-e-d-c-b-a-g-f***=1-2-3-4-5-4-3-2-1, the "unclear interval" is *išartu* and is placed on 2f-4b = ***f-b***

The "unclear interval" in each set gives its name to the set which follows: in *išartu*, the "unclear interval" is *qablītu*. It gives its name to the second ennead: *qablītu*. In this set the "unclear interval" is *nīš tuḫri*. It gives its name to the next ennead: *nīš tuḫri*, and so forth.

All "unclear interval" are pentadic (***f→b***) when the span has seventeen pitches (when the span is restricted, some "unclear intervals" are tetradic, because of inversion) but have different names according to where they are placed. This shows that the enneadic set is a reduction of the Greater Babylonian System (GBS), as CBS 10996 is the reduction of the GBS for a pitch set of seven, or heptad.

Should we take "unclear interval" location numbers as Gurney located them in 1992, where he follows the order of intervals in CBS 10996, they would be either pentads or tetrads. The location of the "unclear interval" in the first part of the text is: 5-2; 1-5; 4-1; 7-4; 3-7; 6-3; 2-6, or (5-1-4-7-3-6). The second part is the inversion of the first part: 5-2; 2-6; 6-3; 3-7; 7-4; 4-1; 1-5, or (2-6-3-7-4-1).

Nomenclature	Approximative pitch	Quantification
1. *niš tuḫrim*	E^4-D^4-C^4-B^3-A^3	$24^{(9:8)}27^{(10:9)}30^{(16:15)}32^{(9:8)}36$
2. *išartum*	D^4-C^4-B^3-A^3-G^3	$27^{(10:9)}30^{(16:15)}32^{(9:8)}36^{(10:9)}40$
3. *embūbum*	C^4-B^3-A^3-G^3-F^3	$30^{(16:15)}32^{(9:8)}36^{(10:9)}40^{(9:8)}45$
4. *nīd qablim*	B^3-A^3-G^3-F^3-E^3	$32^{(9:8)}36^{(10:9)}40^{(9:8)}45^{(16:15)}48$
5. *qablītum*	A^3-G^3-F^3-E^3-D^3	$36^{(10:9)}40^{(9:8)}45^{(16:15)}48^{(9:8)}54$
6. *kitmum*	G^3-F^3-E^3-D^3-C^3	$40^{(9:8)}45^{(16:15)}48^{(9:8)}54^{(10:9)}60$
7. *pītum*	F^3-E^3-D^3-C^3-$B^{3(\flat)}$	$45^{(16:15)}48^{(9:8)}54^{(10:9)}60^{(16:15)}64$

Fig. 12 Quantification of pentads showing that they all differ in content. Quantifications are given in regular numbers and in ratios.

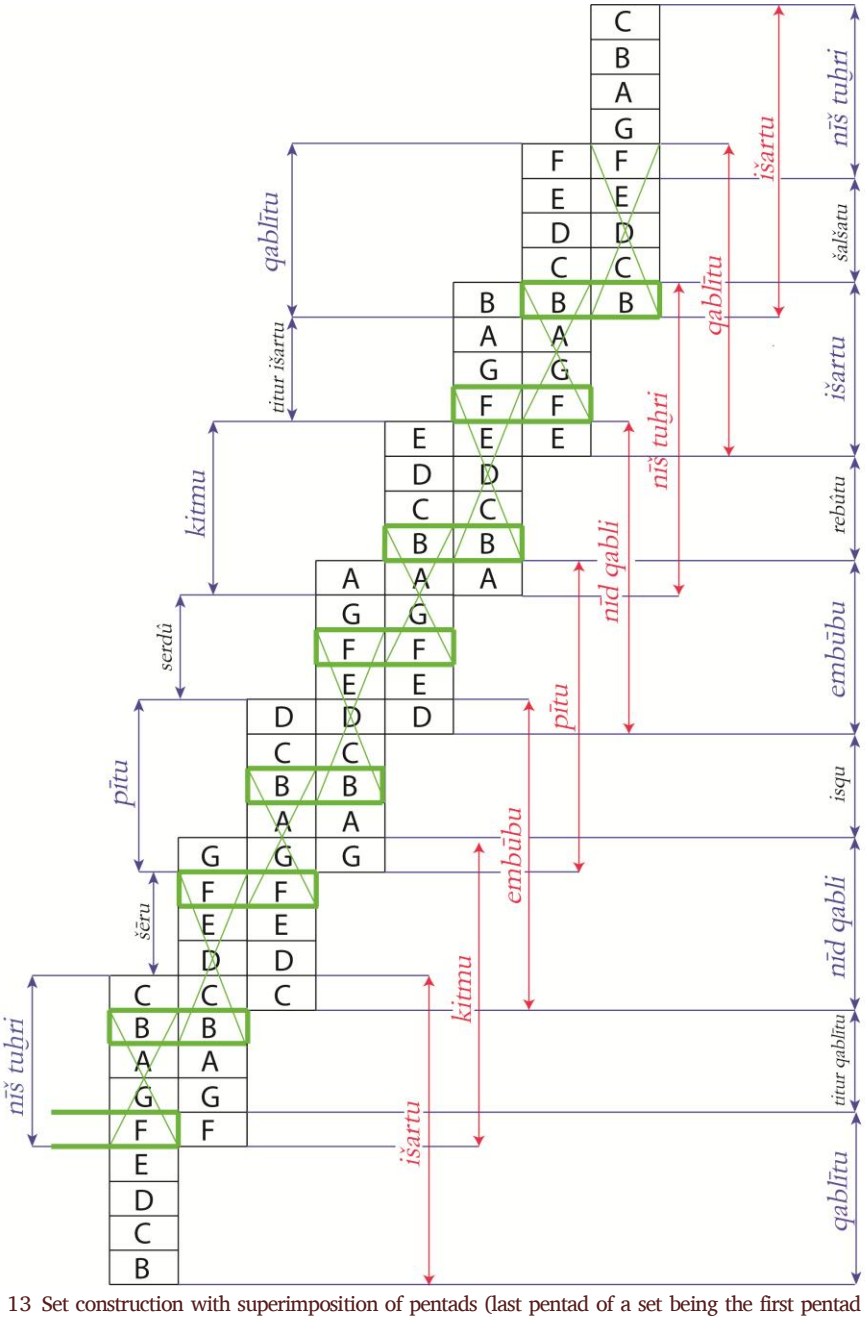


Fig. 13 Set construction with superimposition of pentads (last pentad of a set being the first pentad of the following set). Pentads (principal intervals) are in blue; triads (secondary intervals) are in black; sets are in red. Tritones are crossed with green lines with their conjunction pitch framed in thick green lines.

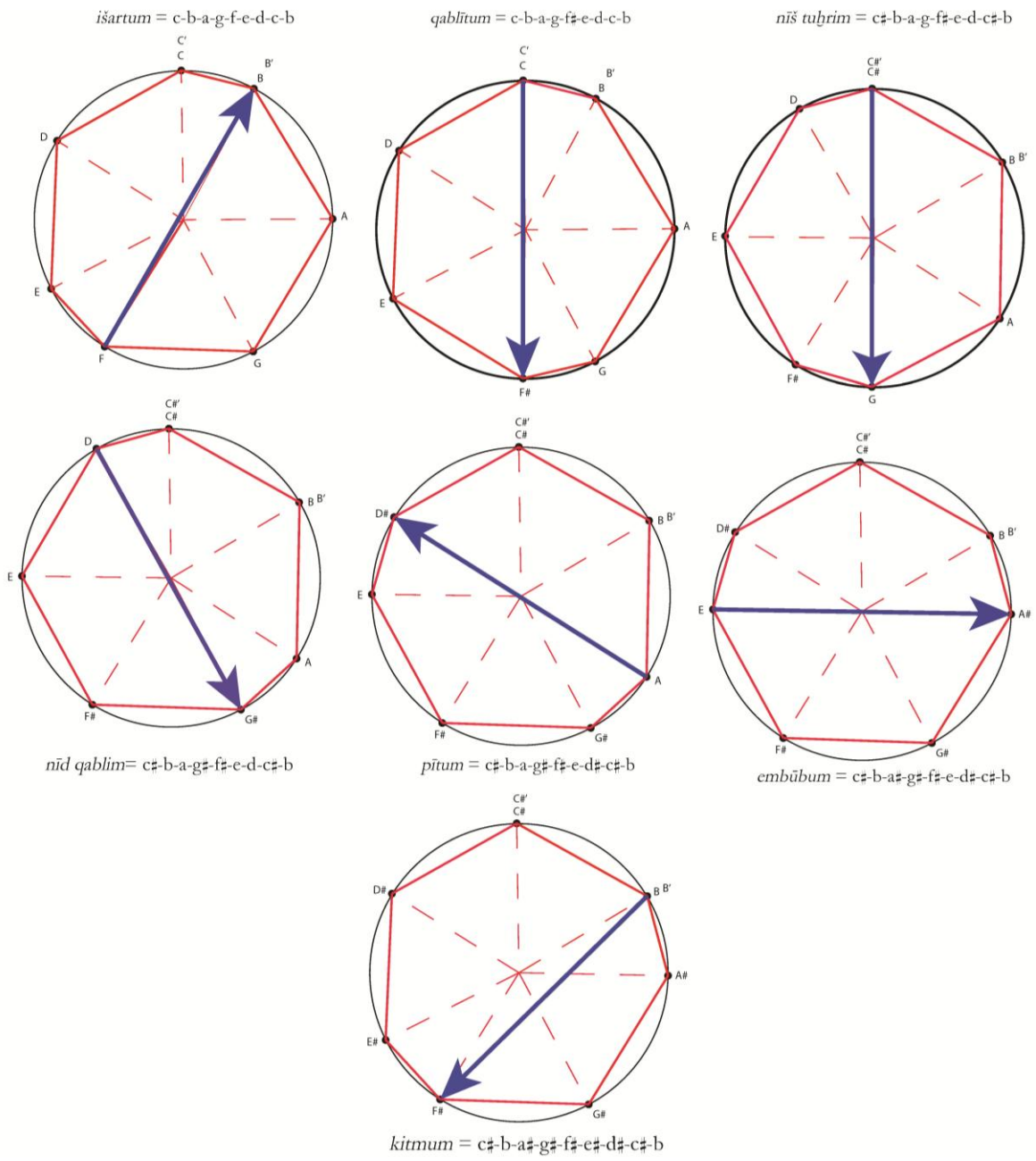


Fig. 14 Circular representation of the sets generated in U.7/80. There is no evidence that Babylonians used this form of representation of their sets mainly because their system was enneatonic. Only heptatonism is suited for its representation in an encircled heptagram. (In my representation, note that I have doubled *C*s and *B*s to fit enneatonism within a heptagram): 1. *išartum*; 2. *qablītu*; 3. *nīš tuḫrim*; 4. *nīd qablim*; 5. *pītum*; 6. *embūbum*; 7. *kitmum*. Blue arrows indicate the tritones and their polarity.

This sequence is exactly the same that we find later with text CBS 1766, (Fig. 16: 114) which, without any doubt is a heptatonic construction. This is how lack of meticulousness and hasty assumptions always lead to wrong conclusions.

If the enneadic sets, which constitute the basis for Babylonian music theory are composed of two conjunct pentadic intervals, then their description and purpose in CBS 10996 has been wrongly interpreted. Therefore, all postulations built from this assumption are consequently flawed.

Philology is only partially understood. My reconstruction of CBS 10996 has correctly positioned the "unclear interval" interval *qablītu* perfectly in the middle of the grid. *pītu* means "opening" and *kitmu* perhaps "closing"; *išartu* means "erect, straight".

All these terms would have had their meanings which at present remain obscure. Usually, various cultures use toponyms to name their scales. Greek theory has Ionian, Dorian, Phrygian, Lydian, Locrian, etc. Babylonian sets also use particular names though not toponymic.

For advocates of heptatonism, I must insist that to prove its existence, there must be evidence for its construction. Without it, the term may not be used. U.7/80 has no evidence of it. The tuning of octaves does not prove that the system is octavial heptatonic. There is incompatibility between the heptatonic system and the octave.[60] A heptatonic set is made up of 6 just tones (5 just tones and 2 semi-tones). A Just tone equals slightly less than 204 cents (9:8) and six of them amount to 1223 (1223.46) cents. The octave measures exactly 1200 cents. In the context of U.7/80, the octave exists as an interval shared between two conjunct pentads but it is not and interval contained within a pentad. Furthermore, a distinction must be made between the octave as an interval and the octave as a concept. These are two very different things. The octave as a concept is a sampling standard within which a certain number of intervals can be fitted for the purpose of measurement. It is a container of smaller intervals[61] as first coined by friend and scholar Amine Beyhom.

A problem remains. How enneadic sets where distinguished from pentadic sets since they have the same names? In 1977, Aaron Schaffer found a small fragment at the University Museum, Philadelphia which he thought might be part of the reverse of *nabnītu* xxxii. The word *siḫpu*, was found associated with each of the enneadic, or pentadic sets:

išartu /siḫip išartu
kitmu/siḫip kitmu
embūbu/siḫip embūbu
pītu/siḫip pītu
nīd qablim/siḫip nīd qablim
nīš tuḫrim/siḫip nīš tuḫrim
qablītum/siḫip qablītum

The order of the sets above corresponds to the second part of the text.

Line 11 of *nabnītu* xxxii is the header of a new list: [sa.]du.a! *pismu*. There, the word *siḫip* precedes sets. Would *pismu* or *siḫip* denominate pentadic or enneadic sets is not possible to say at present.

IV - YBC 11381: *9 SETS?*

A recently published Neo-Babylonian text[62] in the Yale Babylonian Collection, this is one of the most significant additions to the corpus of music theory for the past fifty years.

The text lists nine strings. Each string number is followed by an incipit[63]. The nine strings are known from *nabnītu* xxxii and mentioned in U.7/80. Unlike their disposition in *nabnītu* xxxii where the nine strings are listed palindromically/symmetrically: 1-2-3-4-5-4-3-2-1, YBC 11381, has them listed continuously: 1-2-3-4-5-6-7-8-9, significantly.

Each line start with the Sumerian sign "sa", meaning "string", followed by a number. I believe the nine "sa" with their numbers are no longer used only for listing strings, as with *nabnītu* xxxii, but would also be used for naming nine enneatonic/bi-pentadic sets generated from the system described in text U.7/80.

Interestingly, this new text might be a precursor for Plato's quantification of his nine Muses. In a notoriously difficult passage of *Republic,* (545c-546d)[64] the Muses speak about two harmonies,[65] two Pythagorean heptachords superimposed, Dorian and Phrygian, in such a way that their combination aggregates into an ennead/bi-pentad with pitch quantifications which would have come from the Babylonian model. Clio is *a"* 2400; Euterpe is *g'* 2700; Thaleia, is *f'*, 3000; Melpomene is *e'* 3200; Terpsichore

[60] The term "octave" has been borrowed from Mediaeval Western Christianity. It means a series of eight days preceding a festival. It is contended that by giving the same name to a series of eight notes, it would "Christianize" it thus making of music a religious act. A more appropriate term should be "octade".

[61] [Beyhom, 2003 ; 2010a ; 2013 ; 2017].

[62] [Payne, 2010]. As stated by Payne this tablet can be approximately placed as Neo-Babylonian based on its orthography.

[63] In music, an *incipit* (from the Latin, meaning "it begins") is an initial sequence of notes or words used in catalogues of musical texts.

[64] *e.g.* [Cornford, 1997] omits the description" of Socrates" "sovereign number". In [Plato and Waterfield, 1993] the latter notes "scholars nowadays largely ignore the passage" – see [Crickmore, 2009]: "Hesiod's 'races' and 'political degeneration' in Plato", p. 56-57.

[65] See notes in [Plato and Adam, 1909, v. 2, p. 202–203].

is *d'* 3600; Erato is *c'* 4050; Polyhymnia is *b* 4320; Urania is *a* 4800 and Caliope is *g* 5400. Apart from Erato and Polyhymnia with typical Greek numbers, the other muses have Babylonian quantifications.

The way instructions are given in U.7/80 imply that they would have left room for local, regional, or, and national tone inflections in Old-Babylonian systems allowing for specific intervals to be tuned slightly wider, or slightly smaller than the Just paradigms. They were tuned by ear only, from a master's teachings through metaphors, and metonymy, and not with ratio theories.

Aristoxenos would have preferred speaking in terms of tension (ἐπίτασις) and relaxation (ἄνεσις), but how much of Aristoxenos' works are really his and not Western Mediaeval transpositions of Eastern theories, into Western ones, during and after the crusades, is conjectural.

Al-Fārābī, Latinised as Alfarabius, because of the complete disappearance of the Babylonian cuneiform script would have assumed that most he knew of the past would have mainly come from the Greek: He would have thus lost all knowledge of any Babylonian antecedence, and the Greeks were not eager at giving Babylon any credit.

Since U.7/80 relies only on tension and relaxation of strings, and not with ratios, it follows that the ratio of 2/1, the octave, is irrelevant. Although the "Gurney/Wulstan theory", is interpreted as octavial, tense diatonic heptatonism, all seem to ignore that the first extrapolated scale of *išartum*, is not at the octave of the last *išartum* in both chapters.

It is either higher, or lower, by an unqualified semitone. It would appear logical that the Babylonians, after having adopted an enneatonic generative model, would have had nine subsets. These sets would have stemmed from their fundamental or generative model, as shown below, for the first chapter of U.7/80 with the second chapter relaxing the tension in each set as a reverse process of chapter one.

Note that because we are certain, from the reading of U.7/80, that the last quatrain of the first part and the first of the second was *išartum*, it would be reasonable to assume that the series started with *kitmum*.

Tension (part one)

1	*kitmum:*	*c-b♭-a-g-f-e-d-c-b♭*
2	*išartum:*	*c-b-a-g-f-e-d-c-b*
3	*qablītum:*	*c-b-a-g-f♯-e-d-c-b*
4	*nīš tuḫrim:*	*c♯-b-a-g-f♯-e-d-c♯-b*
5	*nīd qablim:*	*c♯-b-a-g♯-f♯-e-d-c♯-b*
6	*pītum:*	*c♯-b-a-g♯-f♯-e-d♯-c♯-b*
7	*embūbum:*	*c♯-b-a♯-g♯-f♯-e-d♯-c♯-b*
8	*kitmum":*	*c♯-b-a♯-g♯-f♯-e♯-d♯-c♯-b*
9	*išartum":*	*c♯-b♯-a♯-g♯-f♯-e♯-d♯-c♯-b♯*

Relaxation (part 2)

9	*išartum":*	*c♯-b♯-a♯-g♯-f♯-e♯-d♯-c♯-b♯*
8	*kitmum":*	*c♯-b-a♯-g♯-f♯-e♯-d♯-c♯-b*
7	*embūbum:*	*c♯-b-a♯-g♯-f♯-e-d♯-c♯-b*
6	*pītum:*	*c♯-b-a-g♯-f♯-e-d♯-c♯-b*
5	*nīd qablim:*	*c♯-b-a-g♯-f♯-e-d-c♯-b*
4	*nīš tuḫrim:*	*c♯-b-a-g-f♯-e-d-c♯-b*
3	*qablītum:*	*c-b-a-g-f♯-e-d-c-b*
2	*išartum:*	*c-b-a-g-f-e-d-c-b*
1	*kitmum:*	*c-b♭-a-g-f-e-d-c-b♭*

It will be noted that these sets proceed in fifths: *c-g-d-a-e-b-f♯-c♯-g♯*, therefore in the thetic disposition. However, the sets are here in the dynamic disposition (Fig. 15) and that therefore, the eighth set is not the repetition of the first one and the ninth is not the repetition of the second one, one octave higher.

Had the system been octavial, then the first and eighth sets would have been identical and so would have been the second and the ninth.

It is possible that the nine chants listed in the text would have been sung to the following scales:

sa 1 May *Aššur*, the king of the gods, improve your dominion for you.
c-b♭-a-g-f-e-d-c-b♭ = g-f-e-d-c-b-a-g-f = kitmum

sa 2 May *Ištar*, who created mankind, grant you well-being and longevity.
c-b-a-g-f-e-d-c-b = išartum

sa 3 May *Daragal* make you rival the fierce weapon (s and) the raging storm.
c-b-a-g-f♯-e-d-c-b = f-e-d-c-b-a-g-f-e = qablītum

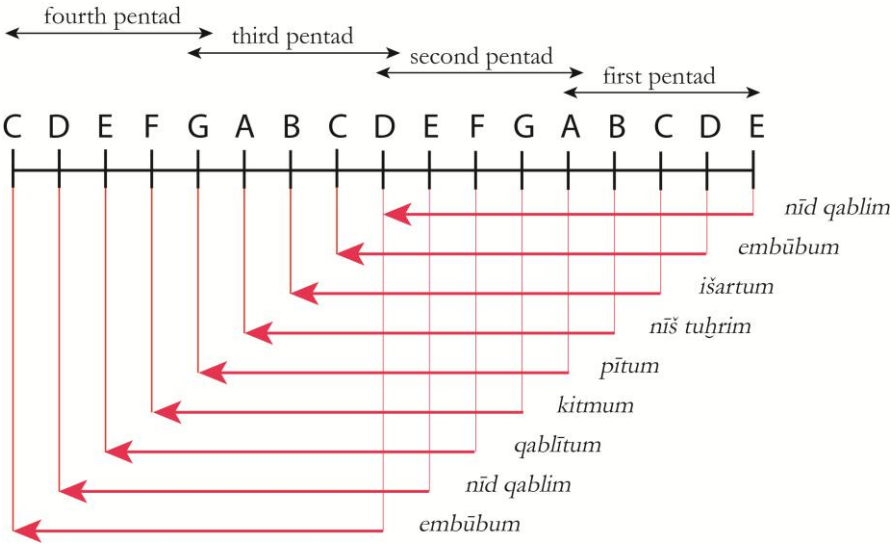


Fig. 15 Placement of the nine sets on the greater Babylonian System of 17 pitches in a dynamic disposition.

sa 4 *Enkidu*, treat kindly the Lady, the protective spirit who created good things, the *lamassu*.
c#-b-a-g-f#-e-d-c#-b = b-a-g-f-e-d-c-b-a = nīš tuḫrim

sa 5 May *Damkianna* make your appeal, your prayers, and the stroke of your nose always pleasing to the lord of lords. *c#-b-a-g#-f#-e-d-c#-b = e-d-c-b-a-g-f-e-d = nīd qablīm*

sa 6 May *Endašurimma* present your artful advice and your precious words daily.
c#-b-a-g#-f#-e-d#-c#-b = a-g-f-e-d-c-b-a-g = pītum

sa 7 May *Endukuga* always let your footstep fall on a prosperous road and a smooth path.
c#-b-a#-g#-f#-e-d#-c#-b = d-c-b-a-g-f-e-d-c = embūbum

sa 8 May *Enudtila* constantly establish abundance, plenty, and prosperity for the pastures of your people.
c#-b-a#-g#-f#-e#-d#-c#-b = g-f-e-d-c-b-a-g-f = kitmum

sa 9 May *Enmešarra* crush the forces of those who wrong you and of your enemies. May he scatter the weapons of your adversaries. *c#-b#-a#-g#-f#-e#-d#-c#-b# = c-b-a-g-f-e-d-c-b = išartum* + undefined quantity

Thus, it is possible that the nine sets were known, at some point during the late Neo-Babylonian period, no longer by their names but by their numbers. There is a parallel in the text which follows (CBS1766) where the names of sets are also substituted by numbers. This also applies, much later to Ecclesiastical Modes such as "mode of the first tone", "mode of the second tone",[66] etc., and no longer by their original Greek names.

V - CBS 1766:[67] Or first evidence of heptatonism

This unusual rectangular tablet dates from the late Neo-Babylonian period, early last half of the first millennium. It has the drawing of an irregular heptagram[68] etched within two concentric circles, at the top left corner, with annotations both lexical and numeral. Under the heptagram, there are eleven columns spreading onto the whole width of the tablet. Columns two and three have seven numbers each. Column four is empty. Columns five, six and seven are inscribed with only one line of numbers. A header

[66] The seven ecclesiastical modes: Mode of the first tone (Ionian) (mode of *c*) *c-d-e-f-g-a-b-c* = 1 1 ½ 1 1 1 ½. Mode of the second tone (Dorian) (mode of *d*) *d-e-f-g-a-b-c-d* = 1 ½ 1 1 1 ½ 1. Mode of the third tone (Phrygian) (mode of *e*) *e-f-g-a-b-c-d-e* = ½ 1 1 1 ½ 1 1. Mode of the fourth tone (Lydian) (mode of *f*) *f-g-a-b-c-d-e-f* = 1 1 1 ½ 1 1 ½. Mode of the fifth tone (Mixolydian) (mode of *g*) *g-a-b-c-d-e-f-g* = 1 1 ½ 1 1 ½ 1. Mode of the sixth tone (Eolian) (mode of *a*) *a-b-c-d-e-f-g-a* = 1 ½ 1 1 ½ 1 1. Mode of the seventh tone (Locrian) (mode of *b*) *b-c-d-e-f-g-a-b* = ½ 1 1 ½ 1 1 1.

[67] [Waerzeggers and Siebes, 2007].

[68] In general, a heptagram is any self-intersecting heptagon, a seven-sided polygon. It is the 7/3 heptagram which is depicted in CBS1766. This is the smallest star polygon which can be drawn in two forms, 7/2 and 7/3, as irreducible fractions.

spreads along the entire length of the columns but at present resists interpretation.

The heptagram and the column two represented in Fig. 16 constitute the first evidence of a heptatonic construction, and therefore of conceptual, if not practical heptatonism. This is based on the names of seven strings inscribed on each point of the heptagram. The names of the strings are given in the same order as they were in *nabnītu* xxxii, without the two last strings (second behind and behind string).

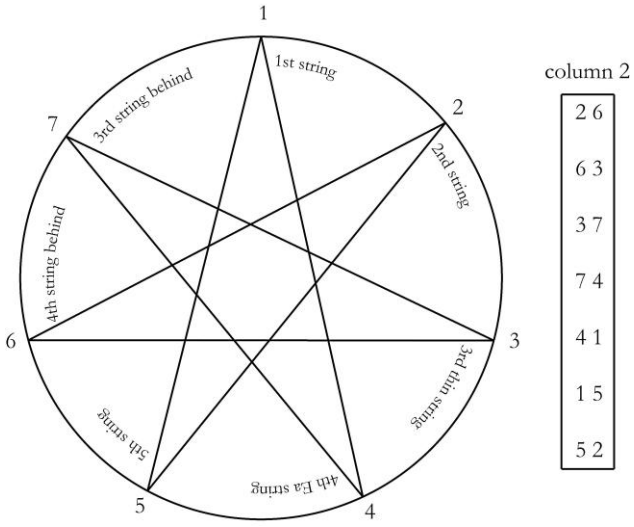


Fig. 16 CBS 1766, selective lexical and numeral translation.

Another significant matter is that the strings are also described with numbers from one to seven. The intersecting lines link the points of the heptagram in a pattern generated by the numbers in column 2: 2-6 = *b-e*; 6-3 = *e-a*; 3-7 = *a-d*; 7-4 = *d-g*; 4-1 = *g-c*; 1-5 = *c-f*; 5-2 = *f-b*. This heptatonic construction consists in the alternation of descending fifths and ascending fourths until the last ascending fourth reaches 5-2 = which should be *f-b♭*, a just fourth. However, the *b♭* would be conflicting with the initial "natural"[69] "*b*". Therefore, to suit the heptagram, the *b♭* is made "natural" so that it equates to the initial "*b*".

This system is radically different in its construction from all previous systems. It indicates, if not a change, but at least an addition to Babylonian theory. While the older system is of linear construction, CBS 1766 is of cyclical expression, two very different concepts. The names of the strings/pitches described in earlier texts are replaced by numbers from 1 to 7.

The system relies only on just fourths and fifths for its construction, and no longer on fifths, fourths and, or thirds. This was a remarkable innovation, typical of Babylonian scholarship that the Greeks would have adopted.

However, it could also be surmised that this innovation was originally Greek and adopted by Babylonian theory during the Orientalizing period, fertile in exchanges between both cultures, but this is highly conjectural as there is no reliable chronological (or other) evidence from the Greek side of the coin. Furthermore, there is no evidence of any heptatonic representation with circumscribed heptagrams in the history of Ancient Greek theory. This theory has always been linear, conceptually and transposed as a tangible monochord on which ratios of string-lengths were applied.

This *modus faciendi* will remain with Boethius, and later theoreticians. It is highly probable that Near-Eastern scholarship adopted the cyclical model which perdured into Islamic theory but was never adopted in the West until much later. Therefore, CBS 1766 might be Babylonian, or perhaps a Babylonian interpretation of a Greek linear model, a hypothesis which I would advance with little conviction.

The Near-East never wilfully adopted the octavial concept and remained attached to smaller intervals known as *ajnās* which agglutinated to one another forming scales. Later, probably with the indoctrination from the crusades, tetrads were added to pentads and triads. There would have been further influence from crusaders who remained in the Levant and probably Westernised local trends. There was further contamination during the French Mandate, in Syria and Lebanon where *Maqām* musicians and teachers were instilled with Western heptatonism.

The belief persists and has greatly contributed to the degeneration of Oriental intonation. But I would advance with conviction that the *Maqām* and other musical forms practiced in the Near-East are direct inheritors of the Babylonian models.

[69] I use the term "natural" reluctantly, as it conveys an impression of normality for the Western tense diatonic scale.

VI - H6: THE "PROOF OF THE PUDDING"?

The Hurrian songs[70] are well documented.[71] Tablet H6 comes from a collection of about thirty which could be restored from three fragments: (RS13.30 + 15.49 + 17.387). The other texts were mostly broken in small fragments which could not be joined. They date from about 1400 BC and were excavated at the site of Ugarit, modern-day Ras-al-Shamrah in North West Syria. The tablets were written in the agglutinative Hurrian language. Fortuitously, musical terms were written in "Hurrianized" Babylonian making it reasonably easy to read. There were hitherto unknown terms the meaning of which remaining obscure. The writing runs parallel to the longest side. It is divided in three. The first part generally spreads onto the obverse. A double line with two winkelhaken[72] is drawn at about the middle of the tablet. Musical notation is written under the double lines.

The musical notation can be segmented in sets and numbers associated with them:

According to Kilmer, the colophon says: [*an-n*]*û za-am-ma-rum ša nid-qib-li za-l*[*u*]*-z*[*i ša* DINGIR.MEŠ TA ^{m}*Urḫiya*] ŠU ^{m}*Am-mu-ra-bi.* This roughly translates as: This is a song in the set of *nid qibli*, a *zaluzi* for the gods, composed by *Urḫiya* and written by the scribe *Am-mu-ra-bi.*

The Chicago Assyrian dictionary says that a *zamāru* is "a song with or without instrumental accompaniment". However, I think that the word "song" is inappropriate because there are instances where the word is used for an *adapu*-instrument. Since an instrument does not sing, I prefer the use of the term melody: "a sung melody", or an "instrumental melody", to avoid confusions. For instance, there is a *za-ma-rum šá pít-ni* which the *CAD* translates as "to sing to the accompaniment of the *pitnu*-instrument". However, this could also translate as "a melody played on the *pitnu*-instrument". In most cases, it is the *ḫalḫallatu*-drum which is mentioned as accompaniment to the voice, but percussion is rhythmical rather than melodic. However, we have instances where *šušqqûssu ina sammî li-iz-za-mir-ma* translates as: "let her (Babylon's) exaltation be sung to the accompaniment of the harp". The only instruments of which we are certain that they accompanied the voice were the *ḫalḫallatu* and the *alû* which are drums; the *balaggu* and the *sammû* are string instruments. My view is that popular instruments such as the pastoral *inu*, a type of primitive lute, would have provided some basic accompaniment to a song. On the other hand, I would think that ritual singing, mostly, would have been "a cappella". The more solemn cultic chants would have had percussion accompaniment.

It took centuries for the Christian Church to adopt the organ. Plain-Chant should have remained unaccompanied because a well-tempered tuned instrument is anathema to ecclesiastic modality. The Babylonian clergy might have had similar views, although not for the same reasons, but it is usually for reasons of spirituality that religious singing mostly remains unaccompanied, in most cultures.

With regard H6, I believe the song did not have any instrumental accompaniment. Had there been, it would have been written down. They certainly could do so. Kilmer's hypothetical accompaniment has been forced-fitted as justification for her belief in simultaneous dyads. The notion of instrumental accompaniment is not a simple matter as it introduces the concept of "absolute" tuning in a world where tuning was "relative".

In the absence of a standard pitch, instrumental accompaniment would have been problematic unless one specific instrument accompanied one specific voice, exclusively. On the other hand, a street or a folk musician could have accompanied him- or herself should the tuning fit with their own "tonal" register, but this certainly would have been exceptional rather than habitual. This problem is of no concern to us, in the West, or at least since the seventeenth century AD, as equal temperament tuning allows for transposition which would certainly not have been possible at Ur or Babylon.

Additionally, the principle of accompaniment, thousands of years ago, was one which would have

[70] I write "songs" in reaction to the sempiternal denomination of any ancient music as "hymn", with a religious connotation. The term "song" means that the melody can be either secular or religious.

[71] [Nougayrol, 1955], [Schaeffer, 1962], [Nougayrol et al., 1968].

[72] The Winkelhaken (from German "angular hook"), also simply called a hook, is one of five basic wedge elements appearing in the composition of signs in Akkadian cuneiform.

involved conceptual understandings for which there is certainly no evidence at that period.

Some of the terms and numbers in Fig. 17 are difficult to read and therefore, the number of beats in the last column to the right reflect these problems. My experience, shouldered by logic, tells me that there must have been a regular infrastructure in this melody. It is written on six lines; it has six intervals per line.

I would confidently guess that the four lines at the centre (2, 3, 4 and 5) amount to thirty-six beats. Therefore, six times six intervals are thirty-six.

I can only but assume that the first and the sixth lines being introductive and conclusive would have twice thirty-six beats, therefore seventy-two beats. However, the rhythmical values of the sets are irregular. This is probably because music had to fit the text and not the contrary.

This suggests that a single melody, whether a song or a hymn, might have different sets of lyrics to accompany and that inevitable metrical variations of the lyrics would be echoed in the time signature for each segment.

The colophon in the text says that this song is in the set of *natqabli*, Babylonian *nīd qablim*, which is: *e-d-c-b-a-g-f-e-d*.

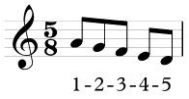
My methodology in this interpretation is as follows (Fig. 17):

- Each set is followed by a number.
- Each line has sets and subsets amounting to six.
- The numbers are rhythmical notation.
- The numbers following the intervals prolong the last beat.
- One number beat equals to two interval beats.

This is the process:

The first cell of the first line is *qablite*. (*qablītu*).

qablite in the set of *nīd qablītu* equals five beats:



...*qablite* followed by 3 equals 5 + (6-1) = 10 beats (5 = beat in the interval. (6-1) 6 is double 3 and -1 is subtracting the last beat of the interval:

My interpretation of the song (Fig. 18) lends itself to analysis. This is a critical point as there is no music without structure.

Here, it is built on the A B C formula. There is an *introduction* at the first line (A), and a *coda* at the last (C).

The *refrain* of the song (B) is composed of four lines of six bars each with a total of 36(?) beats each amounting to six (irregular bars) amounting to a 36/8 time signature, per line split in six bars.

No	I	II	III	IV	V	VI	Beats
1	*qablite 3*	*irbute 1*	*qablite 3*	*šaḫri (?)1(?)*	*titimišarte 10*	*uštamari (?)*	70 (?)
2	*titimišarte 2*	*zirte 1*	*šaḫri 2 (?)*	*šaššate (?) 2*	*irbute 3 (?)*	*šaššate 2 (?)*	38 (?)
3	*umbube 1*	*šaššate 2*	*irbute 3(?)*	*natqabli (?) 1*	*titarqabli 1*	*titimišarte 2 (?)*	38 (?)
4	*zirte 1*	*šaḫri 2*	*šaššate 4*	*irbute 1*	*natqabli 1*	*šaḫri 2*	38 (?)
5	*šaššate 2(?)*	*šaḫri 1*	*šaššate 2*	*šaḫri 1*	*šaššate 2*	*irbute 4(?)*	38 (?)
6	*kitme 2*	*qablite 3*	*kitme 1*	*qablite 4*	*kitme 4 (?)*	*qablite (?) 4(?)*	60 (?)

Fig. 17 H6 notation reconstructed.

Fig. 18 Author's interpretation of Hurrian song of H6. Near-Eastern intonation according to advice from Damascus and implementation with the collaboration of Rosy Azar Beyhom and Amine Beyhom from Beirut. The first bar of the introduction is the fourth bar of the conclusion. It is the musical version of well-known catch-lines often used in Mesopotamian texts. Numbers after accidentals indicate: #1 = 1 comma sharper = 22.64 cents; #2 = 2 commas sharper = 45.28 cents; #3 = 3 commas sharper = 67.92 cents; #5 = 5 commas sharper = 113.2 cents; b1 = 1 comma flat = - 22.64 cents; b2 = 2 commas flat = - 45.28 cents; b3 = 3 comma flat = - 67.92 cents; b4 = 4 comma flat = - 90.57 cents.

The *coda* leads back to the *introduction* to repeat the whole song, as indicated on the tablet with a double winkelhaken on the double bar separating lyrics from music.

However, I am not suggesting that my interpretation is how the piece sounded in 1400 BC. The subsets which I have interpreted, in tense diatonic scale, would have been played with intonations similar to Maqamian *ajnās*.[73] The Babylonian or Hurrian musicians were unable to write down particular intonations for each of their pentads and triads the inflexions of which being as refined as they were complex, and therefore impossible to notate. They still are. However, they were and are inscribed in the memory of the genetic unconscious. Therefore, they had different names. Their recalling, as conditioned reflexes, would immediately suggest how they sounded. Similarly, the accordion evokes Paris; the *ādhān* evokes Cairo; the shofar evokes Jerusalem; pipes evoke Edinburgh, etc. Maqamian *ajnās*, like Babylonian pentads and triads, are called *ajam*, *jiharkah*, *mustaar*, *bayati*, *busalik*, *hijaz*, *kurd*, etc., as Babylonian pentads and triads are called *išartum*, *qablītum*, *šaššate*, *isqu*, etc.

Coranic declamation uses *ajnās* but it must be reminded that these intonations are not specifically Islamic. They were shared by most if not all cultures in the Ancient-East, and continuously throughout history to our days. I am inclined to think that Babylonian music would not have been very different. Hebraic cantillation in Synagogues of Morocco, and Christian music in the Levant, also share these intonations, certainly not as a conscious adoption, or association

[73] A mixed score (Tonogram reproduction of the intonations in parallel with the score) accompanies this article (click here to download in the pdf version), together with a midi reproduction of the intended intonations (here) and an mp3 version of the recording from 2012 with Lara Jokhadar-Aro (here).

with Islamic declamation, but as the reminiscence of an unconscious knowledge.

In Damascus, during the 2011 Oriental Landscapes Conference, I submitted my interpretation of H6 to leading *Maqām* musicians at the Dar al-Assad Opera House. They hummed along my interpretation as it was played. After my presentation, they corrected the melody which I was playing electronically (in the tense diatonic scale), to its proper intonation, and suggested how to play it as it should. These musicians, after over 3000 years, recognized H6 as part of their heritage.[74]

This anecdote is certainly not an academic proof for the authenticity of my interpretation, but it is, certainly, as far as I am concerned, a proof much more significant than any other.

I have titivated the title of this last chapter with the addendum *"proof in the pudding"*. The reason is that my view of Babylonian theory, which clearly diverges from the established version, is consistent throughout, contrarily to Kilmer's. There are no points in this little work which are not fully tested, no more than there are points which do not fit in with the general description of the theory in all texts available to us. The intervals of pentads and triads are the most obvious origins for the Maqamian *ajnās*, and suggest a continuous usage of Babylonian theory from its origins to our days. It is Babylonian music which shaped Early Greek music which in time slowly evolved away from its original model; it is Babylonian music, probably, which shaped some part of Byzantine music;[75] It is Babylonian music which gave the early Mediterranean world musicological tools with which it could, in turn, develop its own concepts.

Music theory was born in Mesopotamia, it was the earliest theory ever developed and is at the source of all other Mediterranean systems and perhaps others.

All music theories of the Ancient Western and Oriental Worlds carry the Babylonian gene and it is therefore not surprising that Plain-Chant modality is so close to it.

[74] A video of this text translated by the author with Oriental adaptation by Rosy Azar Beyhom and Amine Beyhom is available at https://www.youtube.com/watch?v=gynhfxQ1IO4. It must be noted that the singer because of her Western operatic training was unable to give an appropriate Oriental intonation to the piece.

[75] For a rational and comprehensive introspection into Byzantine Music, see [Beyhom, 2015].

Musicological Conclusions

Babylonian music rests on a series of descending pentads and ascending triads with infixes. Two conjunct triads make a pentad and two conjunct pentads make an ennead. The system is essentially descending enneadic, or preferably descending bi-pentadic. Triads, pentads and enneads make up the elements of music similarly to the *ajnās* of the *Maqām* form, of which they are likely to be the source. The sets are organised in systems of enneads, either conjunct when the last pitch of an ennead is the first pitch of the next one, or in organised pentadic conjunction where the last pentad of an ennead is the first pentad of the next one. This is the dynamic arrangement of the system. There is evidence that as early as the Old-Babylonian period the thetic system was also used. It allowed for all sets of a system being contained within a fundamental enneatonic set.

Numbers following pentads and triads indicate the time by which the last pitch of a set should be prolonged. There were other forms which might have been embryonic, and others complementing the system, such as pentatonism and heptatonism, respectively. The ambitus or span of the Greater Babylonian System could expand to 11, 13, 15 and 17 pitches, always arranged in symmetry from the central common pitch, or axis of symmetry. There were nine enneatonic sets as we know from a Neo-Babylonian text, and also from an Old Babylonian tablet which suggests seven enneatonic, but also possibly nine sets. It is probable as with Maqamian *ajnās*, that infixes of pentads were played in any order to suit a composition. Infixes in pentads, while initially diatonic in construction, would have been modified to express mood as with *ajnās*, and like *ajnās*, their names would reflect these variations in intonation.

My exposition of Babylonian music theory radically differs from the established interpretation. However, it is so closely related to Oriental forms, such as the *Maqām*, that it is difficult to ignore this relationship. Ἀπόδοτε οὖν τὰ Καίσαρος Καίσαρι καὶ τὰ τοῦ Θεοῦ τῷ Θεῷ.[76]

[76] "Render unto Caesar" is the beginning of a phrase attributed to Jesus in the synoptic gospels, which reads in full, "Render unto Caesar the things that are Caesar's, and unto God the things that are God's" – [Matthew 22:21].

Ultimate Remarks

The following gems are lifted from Sach's *The Wellsprings of Music*: In his *General History of the Science and Practice of Music*,[77] Sir John Hawkins wrote[78] that "the music of the Barbarians [Orientals] was said to be hideous". Although he studied Greek music he did not realize that the *chroai*, were Oriental. He scorned Oriental music not because it was hideous, but because it was said to be hideous. As for his chapters on the Greeks and the Hebrews, on the contrary, there was no danger of unfavourable reports: conceivably, there were no ear-witnesses.

The two sections of the book were easily filled with learned quotations from literary sources well-known to all the erudite contemporaries. The music itself was absent, to be sure; but being Biblical or Greek, it must have been perfect by definition.

In K.C.F. Krause's unremarkable *Darstellungen aus der Geschichte der Musik*,[79] we read with astonishment that

> "'in Antiquity, which was the childhood of music (!), only simple unadorned melody was known, as is the case today with such peoples as the Hindus, Chinese, Persians, and Arabs, who have not yet progressed beyond the childhood age' (!). This is true Hegelian progressivism: how far have we come in our mature age (or is it senility, if not worse?) (!). Not to mention the profound ignorance behind the notions of Hindus, Persians, and Arabs singing in 'simple unadorned melody' – they who are unrivalled masters in the art of highly adorned singing, and leave simplicity to the lower forms of children's songs – and to the West",[80]

while

> "The reader who reaches for the monumental *Geschichte der Musik* by August Wilhelm Ambros[81] finds a whole *Buch* on the *Kulturvölker des Orients*, indeed on the Primitives. But on these pages, he also finds the most bewildering *pronunciamientos* such as: 'Assyrian music seems never to have risen above the level of a mere sensual stimulus', or: the music of Babylon 'was quite certainly voluptuous, noisy, and far from simple beauty and noble form'; and Phoenician music was mainly meant to drown 'the cries of the victims who burned in the glowing arms of Moloch'".[82]

More recently, Stefan Hagel, in his "Is *nīd qabli* Dorian? – Tuning and modality in Greek and Hurrian music",[83] says:

> "A comparison with Ancient Greek music suggests a largely independent development of musical form at least as early as the first half of the second millennium on".[84]

How could Greeks having borrowed mathematics, astrology, medicine, mythology, religion, divination, literature, law, etc., from the Orient, would have, by some extraordinary phenomenon, forgot all about Babylonian music on their way home?

Further:

> "It is significant that this system was not orientated towards melody, as was Ancient Greek notation and music theory, but to instrumental practice".

What an amazing statement!

Later:

> "For that reason it [the Babylonian system] will survive for a considerable period of time only in a mainly traditional if not backward-orientated musical culture. But in Greek music history the melodic possibilities had soon become too rich to be contained within such a reduced harmonic framework".

What an extraordinary feat of Hellenic supremacism well in keeping with Krause and Ambros. *Plus ça change, plus c'est la même chose.*

[77] [Hawkins, 1776].
[78] For this paragraph: [Sachs, 1962, p. 6].
[79] [Krause, 1827].
[80] Quote and commentary from [Sachs, 1962, p. 7].
[81] [Ambros, 1862].
[82] [Sachs, 1962, p. 7–8].
[83] [Hagel, 2005]: the title is already biased as should it not be: "Is Dorian *nīd qabli*" rather than the contrary?
[84] For this quote and the following: [Hagel, 2005], Abstract, [p. 287].

References

1. AMBROS, August Wilhelm: *Geschichte der Musik 1/*, F.E.C. Leuckart |1862|.
2. Anon. "Gaspard de Prony", *Wikipedia* |2016-10-23| [url: https://en.wikipedia.org/w/index.php?title=Gaspard_de_Prony&oldid=745774476].
3. Anon. "Overtone singing", *Wikipedia* |2017-3-30| [url: https://en.wikipedia.org/w/index.php?title=Overtone_singing&oldid=772916752].
4. BEYHOM, Amine: *Systématique modale – En 3 volumes*, PhD Thesis, Université Paris Sorbonne |2003-9-17| [url: http://www.sudoc.fr/075767538, http://foredofico.org/CERMAA/publications/publications-on-the-site/publications-amine-beyhom].
5. BEYHOM, Amine: "A new hypothesis on the elaboration of heptatonic scales and their origins", *ICONEA 2008*, Lulu on behalf of ICONEA PUBLICATIONS, LONDON-UK |London, 2010a| [url: http://foredofico.org/CERMAA/publications/publications-on-the-site/publications-amine-beyhom] p. 151–209.
6. BEYHOM, Amine: *Théories de l'échelle et pratiques mélodiques chez les Arabes – Volume 1: L'échelle générale et les genres – Tome 1: Théories gréco-arabes de Kindī (IXe siècle) à Ṭūsī (XIIIe siècle) 1/4* (vols.), Librairie orientaliste Paul Geuthner |Paris, 2010b-11|.
7. BEYHOM, Amine: "Un lexique de la modalité", *Near Eastern Musicology Online 2 2* |2013-11| [url: http://nemo-online.org/articles] p. 5–24.
8. BEYHOM, Amine: *Théories et pratiques de l'échelle dans le chant byzantin arabe: Une approche comparative et analytique proposant une solution inédite pour le système théorique de Chrysanthos le Madyte*, Par l'auteur |Broummana (Liban), 2015-9|.
9. BEYHOM, Amine: "A Hypothesis for the Elaboration of Heptatonic Scales", *Near Eastern Musicology Online 4 6* |2017-11| [url: http://nemo-online.org/articles] p. 5–90.
10. CORNFORD, Francis Macdonald: *Plato's Cosmology: The Timaeus of Plato*, Hackett |London, 1997|.
11. CRICKMORE, Leon: "Harmonic Mythology", *Arane I* |2009| p. 53–68.
12. DAC, Pierre: *Les pensées*, Le cherche midi |1981|. (See also http://dicocitations.lemonde.fr/citations/citation-29012.php)
13. DUCHESNE-GUILLEMIN, M.: "Découverte d'une gamme babylonienne", *Revue de Musicologie 49 126* |1963-7-1| [doi: 10.2307/927207. url: http://www.jstor.org/stable/927207] p. 3–17.
14. ELLIS, Alexander John: "On the Sensitiveness of the Ear to Pitch and Change of Pitch in Music", *Proceedings of the Musical Association 3* |1876-1-1| [url: http://www.jstor.org/stable/765234] p. 1–32.
15. FIELD, Hartry: "Epistemological Nonfactualism and the a Prioricity of Logic", *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition 92 1–2* |1998-10-1| [doi: 10.1023/A:1017147216094. url: https://link.springer.com/article/10.1023/A:1017147216094] p. 1–24.
16. FOWLER, David and Eleanor ROBSON: "Square Root Approximations in Old Babylonian Mathematics: YBC 7289 in Context", *Historia Mathematica 25 4* |1998-11-1| [doi: 10.1006/hmat.1998.2209. url: http://www.sciencedirect.com/science/article/pii/S0315086098922091] p. 366–378.
17. GURNEY, Oliver Robert: "An Old Babylonian Treatise on the Tuning of the Harp", *Iraq 30 2* |1968| [doi: 10.2307/4199853] p. 229–233.
18. GURNEY, Oliver Robert: *Middle Babylonian Legal Documents and other Texts, Publications of the joint expedition of the British Museum and of the University Museum of the University of Pennsylvania, to Mesopotamia VII (1973)/*, British Museum Publications |London, 1974|.
19. GURNEY, Oliver Robert: "Babylonian Music Again", *Iraq 56* |1994| [doi: 10.2307/4200387] p. 101–106.
20. HAGEL, Stefan: "Is *nid qabli* Dorian? Tuning and Modality in Greek and Hurrian Music", *BaM 36* |2005| p. 287–348.
21. HAWKINS, John: *A General History of the Science and Practice of Music*, Payne and Son |1776|.
22. HILPRECHT, Hermann Volrath: *Mathematical Metrological and Chronological Tablets from the Temple Library of Nippur*, Department of Archaeology, University of Pennsylvania |Philadelphia, 1906|.
23. KILMER, Anne Draffkorn: "Two New Lists for Mathematical Operations", *Orientalia 29* |1960| p. 273–308 & Tab. LXXXIII.
24. KILMER, Anne Draffkorn: "The Strings of Musical Instruments: Their Names, Numbers, and Significance", *Studies in honor of Benno Landsberger on his seventy-fifth birthday, April 21, 1965*, ed. University of Chicago - Oriental Institute and Benno LANDSBERGER, *Assyriological studies* 16, University of Chicago Press |Chicago, 1965|.
25. KILMER, Anne Draffkorn: "The Discovery of an Ancient Mesopotamian Theory of Music", *PAPS (Proceedings of the American Philological Society) 115 2* |1971| p. 131–149.
26. KILMER, Anne and Sam MIRELMAN: "Mesopotamia", *Grove Music Online* |2001|.
27. KRAUSE, Karl Christian Friedrich: *Darstellungen aus der Geschichte der Musik: nebst vorbereitenden Lehren aus der Theorie der Musik*, Zentralantiquariat d. DDR |1827|.
28. LACAN, Jacques: *Le Séminaire. Encore*, Seuil |Paris, 1975-1-1|. (See also the audio file http://www.valas.fr/IMG/mp3/lacan-encore-20-mars-1973.mp3)
29. LACAN, Jacques: *The Seminar of Jacques Lacan: On Feminine Sexuality, the Limits of Love and Knowledge (Encore) 20/*, Norton &Company |New York, 1999|.
30. LLOYD, Llewelyn S. and Achille FOULD: "International Standard Musical Pitch", *Journal of the Royal Society of Arts 98 4810* |1949| [url: http://www.jstor.org/stable/41364475] p. 74–89.
31. MIRELMAN, Sam: "Tuning Procedures in Ancient Iraq", *Analytical Approaches to World Music 2 2* |2013| [url: http://www.aawmjournal.com/articles/2013b/Mirelman_AAWM_Vol_2_2.pdf] p. 43–56.
32. NOUGAYROL, Jean: "Le Palais Royal d'Ugarit III et Planches", *Mission de Ras Shamra, Tome VI, LE PALAIS ROYAL D'UGARIT III Textes accadiens et hourrites des archives est, ouest et centrales*, ed. Claude Schaeffer |Paris, 1955|.
33. NOUGAYROL, Jean, Claude Frédéric Armand SCHAEFFER, Emmanuel LAROCHE and Charles VIROLLEAUD: *Ugaritica. V: nouveaux textes accadiens, hourrites et ugaritiques des archives et bibliothèques privées d'Ugarit, commentaires des textes historiques (première partie)*, Impr. nationale |1968|.
34. PAYNE, E.: "A New Addition To The Musical Corpus", *Opening the Tablet Box, Near Eastern Studies in Honor of Benjamin R. Foster, Culture and History of the Ancient Near East*, Brill |Leiden, Boston, 2010| [url: http://booksandjournals.brillonline.com/] p. 291–300.

35. PLATO and James ADAM: *The Republic 2/2* (vols.), Cambridge: University Press |1909| [url: http://archive.org/details/republicplat02plat].
36. PLATO and R. WATERFIELD: *Republic*, OUP Oxford |1993|.
37. ROTH, Martha T.: "Oriental Institute | The Assyrian Dictionary of the Oriental Institute of the University of Chicago (CAD)" |2012| [url: http://oi.uchicago.edu/research/pubs/catalog/cad/].
38. SACHS, Curt: *The wellsprings of music*, Martinus Nijhoff |The Hague, 1962|.
39. SCHAEFFER, Claude, ed.: *Nougayrol, Jean, Laroche Emmanuel, Virolleaud, Charles, Schaeffer, Claude, F.A., in Mission de Ras Shamra, Schaeffer, Claude, F.A., Membre de l'Institut, ed. Tome XVI, UGARITICA V, Nouveaux textes accadiens, houssites, et ugaritiques des archives et bibliotheques privees d'ugarit commentaires des textes historiques (Premiere partie) avec la collaboration de Herdner, Andree, Yoyotte, Jean, Aynard, Jean-Marie, Courtois, Leon Chalon-Shaeffer, Isidore de, Szycer, Maurice et de Courtois, Jean-Claude, Secretaire General de l'Edition. Paris, Imprimerie Narionale, Librarie Orientaliste Paul Geuthner, 1868*, *UGARITICA* VI *XV/*, Librairie Orientaliste Paul Geuthner |Paris, 1962|.
40. VITALE, Raoul: "La musique suméro-accadienne, gamme et notation musicale", *Ugarit-Forschungen 9* |1982| p. 241–263.
41. WAERZEGGERS, Caroline and Ronny SIEBES: "An Alternative Interpretation of the Seven-Pointed Star on CBS 1766, NABU 2007/40", *NABU 40 2* |2007-6| [url: https://www.academia.edu/1045063/C._Waerzeggers_and_R._Siebes_An_Alternative_Interpretation_of_the_Seven-Pointed_Star_on_CBS_1766_NABU_2007_40] p. 43–45.
42. WEST, M. L.: *Ancient Greek Music*, Clarendon Press |1992-10-1|.
43. WEST, Martin Litchfield: *Ancient Greek music*, Oxford University Press |1994|.
44. WULSTAN, David: "The Earliest Musical Notation", *Music & Letters 52 4* |1971-10-1| [url: http://www.jstor.org/stable/734711] p. 365–382.
45. YOUNG, Robert W.: "Why an international standard tuning frequency", *The Journal of the Acoustical Society of America 27 2* |1955| [url: http://scitation.aip.org/content/asa/journal/jasa/27/2/10.1121/1.1907908?crawler=true] p. 379–380.